Essays in international finance and macroeconomics

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ESSAYS IN INTERNATIONAL FINANCE AND MACROECONOMICS

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ABSTRACT: *ESSAYS IN INTERNATIONAL FINANCE AND MACROECONOMICS*

The essays contained in this dissertation are described below.

Essay #1: "INTERNATIONAL ECONOMIC POLICY COORDINATION: Policy Analysis in a Staggered Wage-setting Model

The economies of the major industrialized nations have become integrated in many ways, and yet, important structural asymmetries remain between these economies. This paper focuses on one such asymmetry, that of differing wage contracting institutions, and examines the issues of policy coordination and economic interdepedence among such nations within this framework. A two-country model is employed with forward-looking agents who form contracts based on the expected future evolution of price and output. Multi-period dynamic games are played between the policymakers, where both cooperative and noncooperative equilibria are derived for the simulation experiments.

Essay #2: "TESTS FOR LIQUIDITY CONSTRAINTS: A Critique"

Much empirical work has been devoted to estimating the proportion of liquidity constrained consumers (P) and the fraction of income held by these liquidity constrained consumers (L). A common feature of these studies is that P and L are taken to be constant over time. This paper attempts to determine whether this assertion is empirically justified. Using panel data, we find that P and L do vary over time with trend and cyclical components. Hence, this study shows that it is incorrect to estimate P and L as fixed parameters over time, and that P and L should instead be treated endogenously.

Essay #3: "LIQUIDITY CONSTRAINT VOLATILITY: Evidence from Post-War U.S. Aggregate Time-Series Data

The "Life Cycle – Permanent Income Hypothesis" posits the existence of a consumer who chooses a consumption level in each period so as to maximize expected lifetime utility. Credit markets are assumed to be perfect so that the consumer may lend and borrow freely. In the absence of such credit markets, consumers may find themselves unable to satisfy their desired consumption plans, and so, are said to be "liquidity constrained". Using post-war U.S. aggregate time-series data, the importance and persistence of such credit market constraints are examined.

INTRODUCTION

The following three essays address two issues that have gained much recent attention among macroeconomists. The first essay – "International Policy Coordination: Policy Analysis in a Staggered Wage–setting Model" – deals with the incentives for countries to coordinate monetary and fiscal policies in an environment where the countries differ only in the length of the labor contracts which typify their respective economies. The second essay – "Tests for Liquidity Constraints: A Critique" and the third essay – "Liquidity Constraint Volatility: Evidence from Post–war Aggregate Time–series Data" – are tests of the importance and persistence of liquidity constraints in determining consumption behavior in the United States using micro–based data and aggregate time–series data, respectively. Moreover, the latter two essays improve upon the existing literature on liquidity constrained consumption behavior by showing that the degree of these liquidity constraints does vary significantly over time, and so, should be treated endogenously.

The motivation for Essay One comes from the casual observation that the world economy has become more integrated, especially in trade and capital markets, since World War II. It has always been the case over this period that other economies have been significantly affected by economic developments in the United States. In addition, foreign economies have had an increasing role in the economy of the United States, as evidenced by the increasing share of imports and exports in relation to GNP in the United States. In particular, shares of exports as a percentage of GNP have risen from 4.3 in 1950 to 7.0 in 1985, while the shares of imports have risen from 4.1% in 1950 to 10.0% in 1985. This pattern is also shared by the industrialized nations of Europe (such as Germany, France, and the United Kingdom) and in Japan. In Germany, for example, exports as a percentage share of GNP have increased from 11.4 in 1950 to 35.2 in 1985, and imports as a percentage share of GNP have increased from 12.7 in 1950 to 31.3 in 1985. The data unambiguously point to an increased interdependence of the economies of the industrialized world.

This evolution in the world economy has held out the promise of Pareto superior welfare gains to countries through the coordination of their macroeconomic policy decisions. As defined by Wallich [1984], coordination "implies a significant modification of national policies in recognition of international economic interdependence". The increased economic interdependence of national economies requires that policymakers recognize the effects that their policy decisions have on the economies of other nations. Economic theory, in general, supports the notion that Pareto superior welfare outcomes for autonomous nations can be reached through the coordination of monetary and fiscal policies[1], and yet one finds few examples of true policy coordination. As Fischer [1987] points out, "... macroeconomic policy coordination among the major blocs is unlikely to advance beyond the provision of mutual information and occasional agreements for specific policy tradeoffs". Given the apparent absence of explicit policy coordination, it is natural to search for reasons. It is clear that any complete answer involves the political and social, as well as economic characteristics which define these nations. This essay addresses the question by looking at the differences in the very structures of the national economies, most notably, differences in the institutional arrangements of their labor markets.

Although national economies have become integrated and more interdependent in many ways, there remain important structural asymmetries

[1]See Rogoff [1985] and Kehoe [1986] for counter–examples to this principle.

between these economies. One of these asymmetries is the length and staggered nature of the labor contracts which characterize the respective economies. Labor contracts in the United States are generally longer and are renegotiated in a more disaggregated manner (i.e., are more staggered) than in Europe and Japan. These labor market asymmetries have important implications for the views of policymakers regarding the effectiveness of their monetary and fiscal policy decisions. When confronted by adverse economic conditions, different perspectives about policy performance can increase the difficulties of reaching a concensus about the proper policy actions to be taken, and so, complicate their coordination.

Essays Two and Three arise from a common theoretical foundation based on tests for liquidity constrained consumption behavior in the United States.[2] The Life Cycle – Permanent Income Hypothesis (LC–PIH) assumes the existence of individuals who are able to lend and borrow freely against expected future income (i.e., the existence of perfect capital markets) in order to smooth consumption over their lifetime. If individuals are unable to borrow during periods of their lives when income is temporarily low, and so are unable to satisfy their desired consumption plans, they are said to be "liquidity constrained". These credit market imperfections are then the source of liquidity constrained consumption behavior. All previous studies on the presence of liquidity constraints on consumption behavior have treated the estimates of its importance and persistence as being constant over the sample period. Economic theory, however, suggests that the degree of liquidity constraints varies over time. That is, the extent of liquidity constrained consumption

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[2] See Hall [1987] for an overview of the work done on liquidity constrained consumption behavior.

behavior is a function of contemporaneous demographic, credit market, and other economic characteristics. Hence, it should be treated as endogenous. This result is empirically verified in the second essay using a combination of panel data sets – the Survey of Consumer Finance and the Panel Study on Income Dynamics, and in the third essay using aggregate time–series data for the United States. Two measures of the severity of liquidity constrained consumption behavior are used – (i) the proportion of the total population that are liquidity constrained and (ii) the fraction of total income held by liquidity constrained consumers. Both of these measures are used in the second essay, while only the latter is estimated in the third essay. As noted below, it is the latter which is of greater interest to policymakers.

In the second essay, employing a logit estimation technique developed by Jappelli [1987], simulations are performed to derive separate estimates of the fraction of consumers who are liquidity constrained in the years from 1968 to 1982. It is found that these estimated proportions do vary significantly over the sample period. From these estimated fractions of liquidity constrained consumers, approximate estimates are derived for the proportion of total income held by liquidity constrained consumers. It is the latter which is of greatest importance to policymaker, for it is these estimates that have direct application to questions such as the effectiveness tax reform and income redistribution schemes. The second essay is also novel in its use of two separate panel data sets to address the issue of the severity of liquidity constrained consumption behavior.

The third essay employs the insights gained from the second essay by estimating the movement in the fraction of income held by liquidity constrained consumers over the period from 1954 to 1985 using aggregate time–series data. These estimates can be obtained more directly when the estimation is performed using aggregate time–series data. Rather than performing simulations, as was done with the panel data sets in the second essay, the use of aggregate time–series data allows for the estimation of the movement in the fraction of income held by liquidity constrained consumers using contemporaneous economic, demographic, and credit market variables. As in the second essay, the severity of liquidity constrained consumption behavior does vary significantly over the sample period, and is consistent with the general macroeconomic and credit market developments in the United States over the sample period.

As mentioned above, the second and third essays share a common theoretical base. They differ in the empirical methodology employed, but they reach the same conclusion. Future work in this area must acknowledge the fact that the fraction of income held by liquidity constrained consumers should be treated as endogenous.

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INTERNATIONAL ECONOMIC POLICY COORDINATION:

Policy Analysis in a Staggered Wage–Setting Model

Abstract

The economies of the major industrialized nations have become integrated in many ways, and yet, important structural asymmetries remain between these economies. This paper focuses on one such asymmetry, that of differing wage contracting institutions, and examines the issues of policy coordination and economic interdepedence among such nations within this framework. A two–country model is employed with forward–looking agents who form contracts based on the expected future evolution of price and output. Multi–period dynamic games are played between the policymakers, where both cooperative and noncooperative equilibria are derived for the simulation experiments.

SECTION 1: INTRODUCTION

The issues of policy coordination and interdependence among nations have been the focus of much recent theoretical and empirical work. Prolonged episodes of stagnation due to adverse economic shocks and the increasing importance of foreign economies on domestic economic activity have provided strong incentives for such research. The hope has been that concerted action among nations may yield a greater degree of prosperity for all, and yet one finds relatively few instances of policy coordination. Although many forces are at work in the making of policy, such as the unwillingness to relinquish autonomy in policy formation, it is important to look at the economic incentives which nations have to form policy cooperatively.

Serious discussion on the issues of policy coordination must first address the nature of the interdependence with which autonomous nations are confronted. That is, one must consider the economic framework within which policy–making is formulated. Although the economies of the major industrialized nations have increasingly become more integrated, especially in capital markets, there remain significant institutional differences. In particular, one area where there are significant differences between economies is in "wage contracting profiles", i.e., the terms on which wage contracts are formed.

Sachs [1979] has shown that significant institutional differences exist between the United States and the non–North American OECD countries in terms of contract lengths and the staggered nature of contracts. The United States' unionized labor markets are typified by overlapping, long–term wage agreements which are only partially indexed; while Europe and Japan are characterized by some combination of short–term contracts, high indexation, or centralized bargaining.[1] It is these wage–setting institutional differences that have led to differing views about the performance of macroeconomic policy. The greater nominal wage rigidity apparent in the United States has supported the view of a strong link between monetary policy and output among policy–makers, while European and Japanese policy–makers view monetary expansion as primarily feeding inflation. These differing attitudes explain the controversy that existed in the mid–1970's between the United States, Europe and Japan regarding the proper monetary stance in the face of persistent stagflation. This experience of the 1970's and 80's serves to motivate the questions addressed in this paper.

Structural asymmetries can have important effects on the strategic behavior of policy–makers by influencing the economic environment in which policies are formulated. The manner in which monetary and/or fiscal policies of one country affect other countries determines, to a large extent, the optimal policy choice of a country in a strategic setting. This position is supported by Canzoneri and Gray [1983] who posit that the appropriate policy responses for interdependent economies depend on knowing the type of policy regime faced. They consider the (i) "locomotive" case in which a monetary expansion by either country has an expansionary output effect in the other country, (ii) the "beggar–thy–neighbor" case in which a monetary expansion in either country has a contractionary output effect in the other country, and (iii) the "asymmetric" case in which monetary expansions have differing qualitative

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[1] In general, European and Japanese labor contracts are renewed annually; whereas over 70% of the major contract settlements in the United States covered 3 years or more, and less than 2% had settlements of 1 year or less [Sachs (1979)].

(contractionary/expansionary) output effects.

This paper focuses on one particular type of structural asymmetry, that of differing "wage contracting profiles" across countries. The staggered wage–setting structure is "forward–looking" and is similar in spirit to that developed by Calvo [1983]. Wage–setters are assumed to set nominal wages according to the future expected price and output levels over the length of the contract. This is consistent with the Phelps–Taylor staggered contracting approach in that "new" contracts are based upon rational forecasts of the future path of all relevant variables. The forward–looking nature of the wage–setting institutions is important, since this implies that all future demand management policies over the contract are internalized in the current wage level. In a stylized fashion differences in "wage contracting profiles" are introduced via varying contract lengths. Parametric symmetry is otherwise assumed to hold.

The differing nominal rigidities due to asymmetric contract lengths requires that alternative policies be analyzed within a dynamic framework. Although much of the existing literature on policy coordination employs static models in which the policy games are studied, the present paper cannot follow this approach because of the inherently dynamic nature of wage contracting. As a result, policy–makers must not only consider the steady–state effects of a particular policy choice, but also the tradeoffs that exist as the economy adjusts toward a new steady–state. Hence, a dynamic game is better suited to deal with the issues that are central to this paper. Within the given structure, incentives to behave cooperatively versus noncooperative behavior are analyzed. Nash and Stackelberg solutions are the noncooperative equilibria to be considered, while the Nash "bargaining solution" is the cooperative equilibrium to be considered.

The paper is organized as follows. Section 2 discusses the model in detail, including the wage–setting structure that is assumed to exist. Section 3 contains the theoretical derivation of the noncooperative and cooperative equilibria to be considered in this paper. Section 4 discusses the game–theoretic simulation results. Finally, Section 5 concludes the paper with a brief summary of the main results.

SECTION 2: MODEL

A dynamic, two–country, flexible exchange rate, rational expectations model is used. The demand side is explicitly modeled. Supply of the good in each country is assumed to respond sluggishly to excess demand, where each country produces a good that is an imperfect substitute for the good of the other country. Each country is assumed to face two types of stochastic "white noise" shocks: aggregate demand shocks and labor market – supply side shocks. A staggered wage–setting component is included where prices are sticky in the short–run, and where the wage–setting behavior is an important parametric asymmetry between countries. Supply side conditions are not modelled explicitly, but are implicitly characterized by the nominal wage rate of each country and by the adjustment of output to excess demand. Government financing issues are ignored, as are issues dealing with productivity growth over the period under consideration. Regarding the latter, this may be justified by arguing that the horizens of policy–makers are relatively short. The formal model is log–linear expressed as deviations from steady state, where '*' denotes the foreign country.

HOME COUNTRY		FOREIGN COUNTRY
(1)	$y_{t+1} - y_t = \psi_1 [ED_t - y_t]$ where $ED_t = d_1 y^*_t - d_2 [i_t - ((p_{t+1})^\varepsilon - p_t)] + d_3 q_t + g_t + v_{1t}$	$y^*_{t+1} - y^*_t = \psi^*_1 [ED^*_t - y^*_t]$ where $ED^*_t = d_1^* y_t - d_2^* [i^*_t - ((p^*_{t+1})^\varepsilon - p^*_t)] - d_3^* q_t + g^*_t + v_1^*{}_t$
(2)	$w_{t+1} = (1+\gamma) w_t - \gamma (p_t + y_t) - (1+\gamma) v_{2t}$	$w^*_{t+1} = (1+\gamma^*) w^*_t - \gamma^* (p^*_t + y^*_t) - (1+\gamma^*) v_2^*{}_t$
(3)	$p_{t+1} - p_t = \psi_2 (p_{it} - p_t)$	$p^*_{t+1} - p^*_t = \psi_2^* (p_i^*{}_t - p^*_t)$
(4)	$p_{it} = (1 - d_1) w_t + d_1 (w^*_t + e_t)$	$p_i^*{}_t = (1 - d_1^*) w^*_t + d_1^* (w_t - e_t)$
(5)	$m_t - p_t = -a_1 i_t + a_2 y_t$	$m^*_t - p^*_t = -a_1^* i^*_t + a_2^* y^*_t$

(6) $$i_t = i^*_t + [(e_{t+1})^\varepsilon - e_t]$$

(7) $$q_t = p^*_t + e_t - p_t$$

Definition of Variables:

y, y^*: real output ;

w, w^*: nominal wage rate;

p, p^*: general price index;

p_l, p_l^*: long–run price level;

i, i^*: nominal interest rate;

e: nominal exchange rate expressed in terms of the number of units of domestic currency per unit of foriegn currency;

q: real exchange rate expressed as units of domestic goods per unit of the foreign good;

v_1, v_1^*, v_2, v_2^*: "white noise" disturbances.

Equations (1) are the domestic and foreign goods market equilibrium conditions where supply is assumed to adjust to excess demand at rate ψ_1 over time. Aggregate demand in each country is assumed to depend positively on the level of real output in the other country and on its relative competitive advantage (where the Marshall–Lerner condition is assumed to hold), while depending negatively on the real interest rate. As noted by Blanchard [1986], such a situation is plausible if there is some monopoly power in both goods and labor markets. That is, for output to be determined by the level of aggregate demand, one must start from an equilibrium in which price exceeds marginal cost and the wage exceeds the marginal utility of labor.

Equations (2) characterize the movement of nominal wages in the domestic and foreign countries over time. The derivation of (2) is as follows. For each country, it is assumed that contract lengths are stochastic and are independent and identically distributed across contracts. If a contract is in effect at time t, the probability that the contract will last over the next period is given by:

(8) $\quad P[\text{not renegotiated}] = \frac{1}{1+\gamma}$, $\gamma > 0$.

Similarly, the probability that a contract is renegotiated in the next period is:

(9) $\quad P[\text{renegotiated}] = \frac{\gamma}{1+\gamma}$, $\gamma > 0$.

Therefore, if a contract is in effect at time t, the probability that the contract will not be renegotiated for s more periods is:

(10) $$P[s, \gamma] = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{1+\gamma}\right)^s$$

Contract lengths in the economy as a whole are then assumed to be formed according to a geometric distribution and is similar to the continuous–time version presented by Calvo [1983]. The parameters γ (γ^*) neatly characterize the "wage contracting profiles" of the respective countries. Consider the limiting cases:

$$\lim_{\gamma \to 0} P\,(\text{not renegotiated}) = \lim_{\gamma \to 0} [1/(1+\gamma)] = 1$$

$$\lim_{\gamma \to 0} P\,(\text{renegotiated}) = \lim_{\gamma \to 0} [\gamma/(1+\gamma)] = 0$$

$$\lim_{\gamma \to \infty} P(\text{not renegotiated}) = \lim_{\gamma \to \infty} [1/(1+\gamma)] = 0$$

$$\lim_{\gamma \to \infty} P(\text{renegotiated}) = \lim_{\gamma \to \infty} [\gamma/(1+\gamma)] = 1$$

Therefore, the smaller (larger) is γ, the greater (smaller) is the probability that the contract is not renegotiated which implies a longer (shorter) contract length. Assuming homogeneous wage–setting institutions within an individual country who form (the log of) wages rationally taking into account prices and excess demand during the length of the contract yields:

$$(11) \qquad w_t = \frac{\gamma}{1+\gamma} \sum_{i=0}^{\infty} \left(\frac{1}{1+\gamma}\right)^i (p^{\varepsilon}_{t+i} + y^{\varepsilon}_{t+i}) + v_{2t}$$

where $p^{\varepsilon}_{t+i} = p_{t+i}$ and $y^{\varepsilon}_{t+i} = y_{t+i}$ under the rational expectations assumption.

From (11) one is able to derive equations (2).[2] Equations (2) can be rewritten

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[2] Equations (2) are derived from equation (11) in the following manner:

as:

$$w_{t+1} - w_t = \gamma[w_t - p_t - y_t] - (1+\gamma)v_{2t}$$

This form looks strikingly similar to a model in which γ represents an indexing parameter. This is intuitively appealing since shorter contract lengths (i.e., larger values for γ) mean that contracts are negotiated more frequently, and so, is consistent with the indexing interpretation. In addition, larger (smaller) values for γ imply lesser (greater) degree of staggered wage-setting.

Equations (3) and (4) have the general price indices adjusting sluggishly toward a weighted average of domestic and foreign wage costs, where $d_1(d_1^*)$ represent the degree of openness in each country. Beyond the obvious interpretation that residents of one country import goods from the other country, this is also plausible in the presence of a mark-up pricing scheme along with imported intermediate imports. Equations (3) directly account for the effects that nominal wage rates (and so, unit costs of production) have on a country's price level in an open economy setting. The degree of openness of a

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$$w_t = \frac{\left(\frac{\gamma}{1+\gamma}\right)}{1-\left(\frac{1}{1+\gamma}\right)L^{-1}}(p_t + y_t) + v_{2t}$$

$$[(1+\gamma) - L^{-1}]w_t = \gamma(p_t + y_t) + [(1+\gamma) - L^{-1}]v_{2t} ;$$

$$E_t(w_{t+1}) = (1+\gamma)w_t - \gamma(p_t + y_t) - (1+\gamma)v_{2t} + E_t(v_{2,t+1})$$

where L is a lag operator and the transversality condition is assumed to hold. Since $E_t(w_{t+1}) = w_{t+1}$ (by the rational expectations assumption) and $E_t(v_{2,t+1}) = 0$ (since it is a "white noise" disturbance), we have equations (2).

country then determines how exposed that country is to the wage–setting behaviors in the other country. Equations (3) and (4) posit that the general price index in each country adjusts sluggishly to the weighted average of domestic and foreign country unit costs, expressed in units of the country's own currency.

Equations (5) are the money market equilibrium conditions where money demand is negatively related to the interest rate and positively related to the level of real output. It is assumed that residents of one country do not hold the currency of the other country. Equation (6) characterizes the asset market equilibrium condition under perfect capital mobility. Hence, the exchange rate is determined in asset markets under conditions of perfect capital mobility. Finally, Equation (7) defines the real exchange rate in terms of units of the domestic good per unit of the foreign good.

This model differs from many others in that domestic and foreign wages (w, w^*), as well as the nominal exchange rate (e) are nonpredetermined variables which can make discrete jumps in response to "news". This feature leads to important differences in the dynamics of the model as compared to those in which wages are predetermined. With "forward–looking" labor contracts, current and future policy choices are immediately reflected in nominal wages. Longer contract lengths (smaller values of γ) imply that future monetary and/or fiscal policies are more fully internalized in the current nominal wage demands as compared to contracts formed over shorter periods. Hence, policy–makers in countries with longer contract lengths must, to a greater degree, pay "now" for future policy choices. Finally, domestic and foreign prices (p, p^*) and domestic and foreign output (y, y^*) are the predetermined variables of the system.

SECTION 3: DYNAMIC GAMES: The Theoretical Derivations

In the formulation of policy, policymakers understand that their choices influence the evolution over time of the variables to which they attach importance. Likewise, policymakers realize that they are not the only player in this game. Attention must be paid to the spillover effects generated by the policy decisions of other autonomous nations. Hence, policy choices for a country are essentially welfare–optimizing strategic decisions made in an environment in which other autonomous decision–makers are also formulating policy. A dynamic game–theoretic model is therefore the best framework in which to investigate the strategic policy–making of autonomous nations. Both cooperative and noncooperative equilibria are considered. The Nash "bargaining solution" is the particular cooperative equilibrium concept to be considered, while the Nash and Stackelberg solutions are the noncooperative equilibria to be considered.

One observes few occurrences of true cooperative behavior among nations. For example, one notices some degree of policy coordination among nations with some geographic proximity to one another, such as the EEC. In contrast, noncooperative ("competitive") policy–making is the standard "modus operandi" between nations. This is an interesting phenomenon in light of the fact that cooperative solutions are generally Pareto superior to noncooperative solutions.[3] That is, one player (country) may experience a welfare improvement without decreasing the welfare of any other player by adopting a cooperative

[3] Rogoff [1983] analyzes cases in which monetary cooperation between two governments is not Pareto superior to fully–discretionary noncooperative monetary policy.

solution.

Many reasons exist for the failure of nations to form policy cooperatively. A confluence of politico–economic factors have been identified as reasons for this failure. For example, casual observation points to a reluctance of policymakers to relinquish their policy–making autonomy. As noted by Hughes–Hallett [1984], other important considerations include:

i) the gains from cooperation may be insignificant in comparison to the losses in meeting national goals;

ii) cooperative policy–making may redistribute the relative benefits among the nations such that the country which secures the greater benefits in the noncooperative case may not yield as large an improvement in the cooperative case;

(iii) incentives to cheat may exist in the cooperative case such that the welfare loss to the country which maintains its part of the bargain may be too large so that a cooperative agreement is never initiated.

This paper attempts to gain some insight into these disincentives to policy coordination, as well as to investigate the nature of the equilibrium policy feedback rules.

The dynamic games are played over a finite horizon of T periods. Domestic policymakers choose a sequence of policies

$$u = \{ u_1, u_2, \ldots, u_{T-1} \}$$

which minimize their undiscounted quadratic loss function subject to the evolution of the state variables over the T periods of the game:

$$\text{Min} \quad E(W) = \sum_{t=1}^{T} x_t' K x_t$$

$$\text{subject to} \quad x_t = A x_{t-1} + C u_{t-1} + C^* u^*_{t-1} + F v_{t-1}$$

Due to the dynamic structure of the model, policymakers make their policy choices at time t–1 to minimize the quadratic loss function for time t. Put simply, there is a one period lag between the policy choice and its resulting effect upon social welfare. Hence, for the T period game, the policymaker chooses a sequence of T–1 policies. The realized value of the quadratic loss function in the initial period is determined by the assumed initial conditions of the system. Similarly, the foreign country policy makers choose a sequence of policies

$$u^* = \{ u_1^*, u_2^*, \ldots, u_{T-1}^* \}$$

to minimize foreign quadratic losses:

$$\text{Min} \quad E(W^*) = \sum_{t=1}^{T} x_t' K^* x_t$$

$$\text{subject to} \quad x_t = A x_{t-1} + C u_{t-1} + C^* u^*_{t-1} + F v_{t-1}$$

where

$x' = [\ y \ \ y^* \ \ p \ \ p^* \ \ w \ \ w^* \ \ e\ ]$ is the state vector;
$u' = [\ g \ \ m\ ]$ is the vector of domestic instruments;
$u^{*\prime} = [\ g^* \ \ m^*\ ]$ is the vector of foreign instruments;
$v' = [\ v_1 \ \ v_2 \ \ v^*_1 \ \ v^*_2\ ]$ is a vector of domestic and foreign shocks;
A_{7x7} matrix, C_{7x2} matrix, C^*_{7x2} matrix, F_{7x4} matrix;
K_{7x7} and K^*_{7x7} are non–negative definite symmetric matrices.

The constraint for each policymaker's optimizatioon problem defines a first–order discrete time system in which only the first lag of each endogenous variable, policy variable and random shock influences the current value of each endogenous variable. Equations (1) through (7) can be rewritten to yield the above matrix form for the constraint, which is identical for each policymaker. The A matrix captures the effect of last period's state variables on the level of the current state variables, i.e., it expresses the internal evolution of the state variables over time. The C (C*) matrix captures the effects of last period's domestic (foreign) policy choices on the current level of the state variables. The F matrix embodies the effects of last periods stochastic shocks upon the current level of the state variables.

The domestic policymaker attempts to minimize the deviations of domestic output and domestic price level from their trend values by the appropriate choice of domestic policy instruments, while the foreign policymaker chooses policy so as to minimize the deviations of foreign output and foreign price level from trend. The K and K* matrices are essentially the "weighting matrices" for the domestic and foreign countries which identify the state variables which are of importance to the respective policymaker.

In this paper,

$$K = \begin{bmatrix} \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad K^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\beta^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $0 \leq \beta, \beta^* \leq 1$.

The weights β (β^*) are assigned to the domestic (foreign) output deviations, and $1-\beta$ ($1-\beta^*$) are assigned to the domestic (foreign) general price level deviations.

Dynamic programming is used to solve this finite horizon constrained minimization problem. The solution is recursive in that the final period optimization problem is first solved, and then by a process of backward recursion one is able to solve for all periods of the problem. The advantage, as noted by many authors, is that the multi–period optimization problem is reduced to a series of one–period optimization problems. Having solved for the optimal policy rules in the final period, one moves back to the second–to–last period and solves the optimization for that period, conditional upon the policy moves made in the last period. Hence, in each period, the current period's optimization problem is solved, taking into account the optimal policy rules made for all future periods of the game.

A. Feedback Nash Equilibrium

Under the feedback Nash equilibrium, the policymaker in each country, in deriving his own policy rule, assumes that the other country's policy rule is invariant to changes in his own policy choices, i.e., the domestic policymaker believes that $\left[\frac{\partial u^*}{\partial u}\right] = 0$ and the foreign policymaker believes $\left[\frac{\partial u}{\partial u^*}\right] = 0$.

In their general form, the Nash equilibrium results are symmetric in the sense that the foreign country results may be obtained from the domestic country results simply by inserting the foreign country matrices for their domestic analogs. Hence, only the domestic country equations are formally derived here. The recursive solution for any period T–i is given below. See Appendix A for the formal derivation of the general recursive forms.

Period T–i:

Solving backward over the T–1 periods in which the optimal policy rules are formed, one can determine a recursive solution for any period T–i, i = 1,...,T. In period T–i, the domestic policymaker minimizes the period (T–i+1) quadratic loss function:

$$\min_{\{u_{t-i}\}} \quad V_{T-i+1} = x'_{T-i+1} K x_{T-i+1} + E_{T-i}(\bar{V}_{T-i+2})$$

$$\text{subject to} \quad x_{T-i+1} = A x_{T-i} + C u_{T-i} + C^* u^*_{T-i} + F v_{T-i}$$

where

$$E_{T-i}(\bar{V}_{T-i+2}) = x'_{t-i+1} A' [\Omega^{N\,'}_{T-i+1} D^{N}_{T-i+1} \Omega^{N}_{T-i+1}] A x_{t-i+1}$$

since $E_{T-i}(v_{T-i+1}) = 0$; and where $\bar{V}_{T-i+2}$ is the "cost–to–go" term which represents the optimized value of the social welfare functions from period T–i+2 to period T. D and Ω are defined below.

By substitution, the optimization problem can be rewritten as:

$$\underset{\{u_{T-i}\}}{\text{Min}} \quad [\, A\, x_{t-i} + C\, u_{T-i} + C^{*}\, u^{*}_{T-i} + F\, v_{T-i}\,]'\, D^{N}_{T-i}\, [\, A\, x_{t-i} + C\, u_{T-i} + C^{*}\, u^{*}_{T-i} + F\, v_{T-i}\,]$$

$$\text{where } D^{N}_{T-i} = K + A'\,[\,\Omega^{N\,'}_{T-i+1}\, D^{N}_{T-i+1}\, \Omega^{N}_{T-i+1}\,]$$

The resulting first-order condition $\left(\dfrac{\partial V_{T-i+1}}{\partial u_{T-i}} = 0 \right)$ yields the domestic country reaction function:

$$u_{T-i} = -\,G^{N}_{T-i}\,[\, A\, x_{T-i} + F\, v_{T-i} + C^{*}\, u^{*}_{T-i}\,]$$

$$\text{where } G^{N}_{T-i} = [\, C'\, D^{N}_{T-i}\, C\,]^{-1}\, C'\, D^{N}_{T-i}$$

The corresponding foreign country reaction function is:

$$u^{*}_{T-i} = -\,G^{*N}_{T-i}\,[\, A\, x_{T-i} + F\, v_{T-i} + C\, u_{T-i}\,]$$

$$\text{where } G^{*N}_{T-i} = [\, C^{*\prime}\, D^{*N}_{T-i}\, C^{*}\,]^{-1}\, C^{*\prime}\, D^{*N}_{T-i}$$

$$D^{*N}_{T-i} = K^{*} + A'\,[\,\Omega'_{T-i+1}\, D^{*N}_{T-i+1}\, \Omega_{T-i+1}\,]\, A$$

The domestic and foreign Nash equilibrium feedback policy rules are:

$$u_{T-i} = -\left(R^{N}_{T-i} \right)^{-1} S^{N}_{T-i} \left[A\, x_{T-i} + F\, v_{T-i} \right]$$

where $$R^{N}_{T-i} = I - \{ [G^{N}_{T-i} C^{*}] [G^{*N}_{T-i} C] \}$$

$$S^{N}_{T-i} = G^{N}_{T-i} \left[I - C^{*} G^{*N}_{T-i} \right]$$

and

$$u^{*}_{T-i} = -\left(R^{*N}_{T-i} \right)^{-1} S^{*N}_{T-i} \left[A\, x_{T-i} + F\, v_{T-i} \right]$$

where $$R^{*N}_{T-i} = I - \{ [G^{*N}_{T-i} C] [G^{N}_{T-i} C^{*}] \}$$

$$S^{*N}_{T-i} = G^{*N}_{T-i} \left[I - C\, G^{N}_{T-i} \right]$$

The Nash equilibrium values of the period T–i objective functions are then:

$$\bar{V}_{T-i+1} = \left(A\, x_{T-i} + F\, v_{T-i} \right)' \left[\Omega^{N\,'}_{T-i} D^{N}_{T-i} \Omega^{N}_{T-i} \right] \left(A\, x_{T-i} + F\, v_{T-i} \right)$$

where $$\Omega^{N}_{T-i} = I - C \left(R^{N}_{T-i} \right)^{-1} S^{N}_{T-i} - C^{*} \left(R^{*N}_{T-i} \right)^{-1} S^{*N}_{T-i}$$

$$\bar{V}^{*}_{T-i+1} = \left(A\, x_{T-i} + F\, v_{T-i} \right)' \left[\Omega^{N\,'}_{T-i} D^{*N}_{T-i} \Omega^{N}_{T-i} \right] \left(A\, x_{T-i} + F\, v_{T-i} \right)$$

B. Feedback Stackelberg Equilibrium

Unlike the Nash equilibrium, the Stackelberg solution introduces differences in the way that each player accounts for the other player's policy reaction. Under the Nash game they were identical, i.e., each assumed that the other player did not react at all. The Stackelberg game posits two types of players: a "leader" and a "follower". The Stackelberg equilibrium has much to offer in terms of its real world relevance. In the absence of overt cooperation, the assumption that smaller countries form policy, to some degree, in response to policy moves by more dominant players is frequently observed. Here we assume that the foreign country is the "follower" and the domestic country is the "leader". The "follower" forms his conjectures about the "leader's" policy reaction to his own policy moves as in the Nash game, i.e., $(\delta u/\delta u^*)=0$. However, the "leader" internalizes the "follower's" policy reaction function into the derivation of his own optimal policy rule. Hence, the "leader's" optimal policy rule is a function of the current state variables (x), the current disturbances (v), and the policy reaction function of the "follower":

$$u_t = f(\, x_t,\, v_t,\, u_t^*(x_t, v_t, u_t)\,)$$

The Stackelberg equilibrium implies that within the intertemporal dynamics of the game considered here, there exists a "leader"–"follower" sequence of actions within each particular period. Consequently, the general form of the optimal feedback rules under the Stackelberg game are not symmetric as in the Nash game.

Again a solution for the Stackelberg equilibrium is derived via a process of backward recursion beginning with the final period. This is done formally in

Appendix A. From this one can determine the general form of the solution for any period T–i.

Period T–i

The foreign country's ("follower's") optimization problem in period T–i is identical to its Nash problem. Hence, the follower's reaction function is:

$$(12) \qquad u^*_{T-i} = -G^{*S}_{T-i}\,[\,Ax_{T-i} + F\,v_{T-i} + C\,u_{T-i}\,]$$

where $G^{*S}_{T-i} = [\,C^{*\prime}\,D^{*S}_{T-i}\,C^*\,]^{-1}\,C^{*\prime}\,D^{*S}_{T-i}$

$$D^{*S}_{T-i} = K^* + A'\,[\,\Omega^{*S\,\prime}_{T-i+1}\,D^{*S}_{T-i+1}\,\Omega^{*S\,\prime}_{T-i+1}\,]\,A$$

Now consider the domestic country's ("leader's") optimization problem in period T–i:

$$\underset{\{u_{T-i}\}}{\text{Min}} \quad V_{T-i+1} = x'_{T-i+1}\,K\,x_{T-i+1} + E_{T-i}\,(\bar{V}_{T-i+2})$$

subject to $\quad x_{T-i+1} = A\,x_{T-i} + C\,u_{T-i} + C^*\,u^*_{T-i} + F\,v_{T-i}$

$$E_{T-i}\,(\bar{V}_{T-i+2}) = x'_{T-i+1}\,A'\,[\,\Omega^{S\,\prime}_{T-i+1}\,D^{S}_{T-i+1}\,\Omega^{S}_{T-i+1}\,]\,A\,x_{T-i+1}$$

and equation (12).

Substituting the constraints as well as the foreign country's reaction function into the domestic objective function, the optimization problem becomes:

$$\underset{\{u_{T-i}\}}{\text{Min}} \quad [A x_{T-i} + F v_{T-i} + C u_{T-i}]' \{[I - C^* G^{*S}_{T-i}]' D^S_{T-i} [I - C^* G^{*S}_{T-i}]\} [A x_{T-i} + F v_{T-i} + C u_{T-i}]$$

where $D^S_{T-i} = K + A' [\Omega^{S\,'}_{T-i+1} D^S_{T-i+1} \Omega^S_{T-i+1}] A.$

In its general form the first–order condition is:

$$\frac{\partial V_{T-i+1}(x_{T-i}, v_{T-i}, u_{T-i}, u^*_{T-i}(x_{T-i}, v_{T-i}, u_{T-i}))}{\partial u_{T-i}} = 0$$

The optimal policy rule for the Stackelberg "leader" is then:

$$u_{T-i} = -(R^S_{T-i} C)^{-1} R^S_{T-i} [A x_{T-i} + F v_{T-i}]$$

where $R^S_{T-i} = C' [I - C^* G^{*S}_{T-i}]' D^S_{T-i} [I - C^* G^{*S}_{T-i}]$

Substituting the "leader's" optimal policy rule into the "follower's" reaction function, the "follower's" optimal policy rule is:

$$u^*_{T-i} = -S^{*S}_{T-i} [A x_{T-i} + F v_{T-i}]$$

where $S^{*S}_{T-i} = G^{*S}_{T-i} \{I - C (R^S_{T-i} C)^{-1} R^S_{T-i}\}$

The optimized period T–i domestic and foreign objective functions are then:

$$\bar{V}_{T-i+1} = (A\, x_{T-i} + F\, v_{T-i})' \, [\Omega^{S\,'}_{T-i} D^{S}_{T-i} \Omega^{S}_{T-i}] \, (A\, x_{T-i} + F\, v_{T-i})$$

where $\Omega^{S}_{T-i} = [I - C^{*} G^{*S}_{T-i}] \, [I - C\,(R^{S}_{T-i} C)^{-1} R^{S}_{T-i}]$

$$\bar{V}^{*}_{T-i+1} = (A\, x_{T-i} + F\, v_{T-i})' \, [\Omega^{*S\,'}_{T-i} D^{*S}_{T-i} \Omega^{*S}_{T-i}] \, (A\, x_{T-i} + F\, v_{T-i})$$

where $\Omega^{*S}_{T-i} = [I - C\,(R^{S}_{T-i} C)^{-1} R^{S}_{T-i} - C^{*} S^{*S}_{T-i}]$

C. Cooperative Frontier

Under the cooperative game, domestic and foreign policymakers coordinate their policy choices so as to minimize a single quadratic loss function which is a weighted average ($0 \leq \alpha \leq 1$) of their individual quadratic loss functions. The scalar "α" denotes the relative weight given the domestic objective as compared to the foreign objective in the cooperative policy problem. Given a convex opportunity set, the choice of the parameter "α" will isolate a unique point on the cooperative frontier which bounds the opportunity set. In general, the advantage, from a welfare standpoint, of cooperative policymaking is that policy tradeoffs are permitted here (in contrast to noncooperative games) which may increase the welfare of one (or both) countries, with no country being worse off. This is one explanation for the standard result that such cooperative solutions are generally Pareto superior to noncooperative solutions. And yet, as mentioned above, there may exist disincentives which inhibit a particular country from engaging in cooperative policymaking.

Again, the general form cooperative problem is presented here for any period T–i. The more formal derivation is performed in Appendix A.

Period T–i

The single constrained minimization problem for period T–i can be initially expressed as:

$$\min_{\{u_{T-i},\, u^*_{T-i}\}} \quad V_{T-i+1} = x'_{T-i+1} [\alpha K] x_{T-i+1} + x'_{T-i+1} [(1-\alpha) K] x_{T-i+1} + E_{T-i} (\bar{V}_{T-i+2})$$

subject to $x_{T-i+1} = A x_{T-i} + C u_{T-i} + C^* u^*_{T-i} + F v_{T-i}$

$$E_{T-i} (\bar{V}_{T-i+2}) = x'_{T-i+1} A' [\Omega^{C\,'}_{T-i+1} (\alpha D^{C}_{T-i+1} + (1-\alpha) D^{*C}_{T-i+1}) \Omega^{C}_{T-i+1}] A x_{T-i+1}$$

By substitution of the the constraints into the objective function and redefining terms, this can be rewritten as:

$$\min_{\{u_{T-i},\, u^*_{T-i}\}} \quad [A x_{T-i} + C u_{T-i} + C^* u^*_{T-i} + F v_{T-i}]' \; H_{T-i} \; [A x_{T-i} + C u_{T-i} + C^* u^*_{T-i} + F v_{T-i}]$$

where $H_{T-i} = [\alpha D^{C}_{T-i} + (1-\alpha) D^{*C}_{T-i}]$

$$D^{C}_{T-i} = K + A' [\Omega^{C\,'}_{T-i+1} D^{C}_{T-i+1} \Omega^{C}_{T-i+1}] A$$

$$D^{*C}_{T-i} = K^* + A' [\Omega^{C\,'}_{T-i+1} D^{*C}_{T-i+1} \Omega^{C}_{T-i+1}] A$$

The first-order conditions with respect to u_{T-i} and u^*_{T-i} are:

$$u_{T-i} = - G^{C}_{T-i} [A x_{T-i} + F v_{t-i} + C^* u^*_{T-i}]$$

where $G^{C}_{T-i} = [C' H_{T-i} C]^{-1} C' H_{T-i}$

$$u^*_{T-i} = -G^{*C}_{T-i}\,[\,A\,x_{T-i} + F\,v_{t-i} + C\,u_{T-i}\,]$$

where $$G^{*C}_{T-i} = [\,C^{*\prime}\,H_{T-i}\,C^*\,]^{-1}\;C^{*\prime}\,H_{T-i}$$

The domestic and foreign policy reaction functions may be solved simultaneously to reveal the domestic and foreign optimal policy rules for period T–i:

$$u_{T-i} = -(\,R^{C}_{T-i}\,)^{-1}\;S^{C}_{T-i}\,[\,A\,x_{T-i} + F\,v_{T-i}\,]$$

where $$R^{C}_{T-i} = I - [G^{C}_{T-i}\,C^*\,]\,[G^{*C}_{T-i}\,C\,]$$

$$S^{C}_{T-i} = G^{C}_{T-i}\;[\,I - C^*\,G^{*C}_{T-i}\,]$$

$$u^*_{T-i} = -(\,R^{*C}_{T-i}\,)^{-1}\;S^{*C}_{T-i}\,[\,A\,x_{T-i} + F\,v_{T-i}\,]$$

where $$R^{*C}_{T-i} = I - [G^{*C}_{T-i}\,C\,]\,[G^{C}_{T-i}\,C^*\,]$$

$$S^{*C}_{T-i} = G^{*C}_{T-i}\;[\,I - C\,G^{C}_{T-i}\,]$$

The optimized objective function is then obtained by substituting the domestic and foreign optimal policy rules into the original objective function, which yields:

$$\bar{V}_{T-i+1} = (A\, x_{T-i} + F\, v_{T-i})' \, [\Omega^{C\,'}_{T-i} H_{T-i}\, \Omega^{C}_{T-i}] \, (A\, x_{T-i} + F\, v_{T-i})$$

where $$\Omega^{C}_{T-i} = [I - C (R^{C}_{T-i})^{-1} S^{C}_{T-i} - C^* (R^{*C}_{T-i})^{-1} S^{*C}_{T-i}]$$

Since $H_{T-i} = aD_{T-i} + (1-a)D^*_{T-i}$, the minimized levels of the domestic and foreign quadratic loss functions can be identified from the above equation:

$$\bar{V}_{d,T-i+1} = (A\, x_{T-i} + F\, v_{T-i})' \, [\Omega^{C\,'}_{T-i} (\alpha\, D^{C}_{T-i})\, \Omega^{C}_{T-i}] \, (A\, x_{T-i} + F\, v_{T-i})$$

$$\bar{V}_{f,T-i+1} = (A\, x_{T-i} + F\, v_{T-i})' \, [\Omega^{C\,'}_{T-i} ((1-\alpha)\, D^{*C}_{T-i})\, \Omega^{C}_{T-i}] \, (A\, x_{T-i} + F\, v_{T-i})$$

D. Nash fixed threat bargaining model [4]

Two–player cooperative games are considered here. Cooperative games differ from noncooperative games in that the former allow for binding agreements. The particular cooperative agreement employed in this paper is the Nash fixed threat bargaining model [Nash(1950)]. The focus of this section will be to outline the fundamental aspects of this cooperative game.

Denote $H \subset \Re^2$ as the set of feasible payoff pairs for each player and $d \in H$ as the "threat point". If $u = (u_1,u_2) \in H$, then there exist (joint) actions available to the players that result in the payoff u. Otherwise, in the absense of coordinated action, $d = (d_1,d_2)$ is the resulting payoff. No scope for further arbitration is assumed to exist. The pair $\Gamma = (H,d)$ is defined to be a two–player

[4] See James W. Friedman [1986].

fixed threat bargaining game if $H \subset \Re^2$ is compact and convex, $d \in H$, and H contains at least one element, u, such that $u >> d$. Two other definitions are necessary. The set of two–player fixed threat bargaining games is denoted W. Finally, a solution to $(H,d) \in W$ is a function f(H,d) that associates a unique element of H with the game $(H,d) \in W$, where $f(H,d) = (f_1(H,d), f_2(H,d))$.

The conditions defining the Nash solution for this game are:

(i) *individual rationality* : the solution payoff to each player should yield a welfare level at least as large as the player would receive if no agreement were reached;

(ii) *invariance* to positive affine utility transformations;

(iii) *symmetry* : if the feasible set H is symmetric about a 45° line through the origin and $d_1 = d_2$, then $f_1(H,d) = f_2(H,d)$;

(iv) *independence of irrelevant alternatives* : if (H,d), $(H',d') \in W$, $d = d'$, $H \subset H'$, and $f(H',d') \in H$, then $f(H,d) = f(H',d')$.

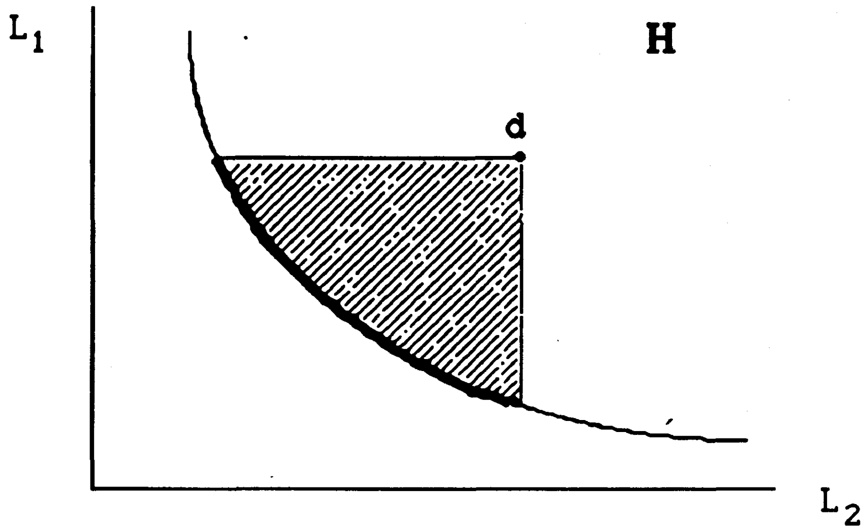
Conditions (i) through (iv) imply that f(H,d) is Pareto optimal.

The Nash "bargaining" solution is that element H which maximizes the product ("Nash product") of gains from agreement. Recalling that policymakers attempt to minimize a quadratic loss function in order to maximize welfare, the problem becomes one of maximizing:

$$(d_1 - L_1) * (d_2 - L_2)$$

where L_i is the quadratic loss value for the ith player (i = 1,2). Figure 1 illustrates the general case for the Nash "bargaining" problem.

Figure 1



The frontier of the set H represents the cooperative frontier. Movements to the NW along the frontier imply a larger weight given to player 2's objective in the cooperative agreement (i.e., a lower α) while movements to the SE mean larger weights are given to player 1's objective (i.e., larger α). The shaded area of H is the region of outcomes that are individually rational, referred to as the *negotiation set.* . Outcomes for the "bargaining" game may then be restricted to the negotiation set. Furthermore, feasibility and Pareto optimality require that the "bargaining" outcome lie along the cooperative frontier. Hence, the "bargaining" problem reduces to choosing a point (or choosing a value of α) along the cooperative frontier within the negotiation set (shown as the dark section of the cooperative frontier in Figure 1). In the absence of a cooperative agreement, each player receives the "threat point" outcome, which is assumed to be the Nash "noncooperative" equilibrium point.

SECTION 4: RESULTS

In this section we study the qualitative effects of asymmetric "wage contracting profiles" on the dynamic game results using a series of policy simulation experiments. The game–theoretic equilibria derived in the preceding section are embedded in the stylized model to address a number of issues:

What is the effect of differing "wage contracting profiles" on the welfare levels of the respective countries?

What effect does the choice of the policymakers' time horizon have upon the evolution and result of the dynamic games?

What impact do labor market asymmetries have on the incentives to form policy cooperatively?

Do dominant strategies (i.e.,policy plays) exist for particular players?

Do Stackelberg "leader–follower" advantages exist for the players?

A Fiscal (Monetary) game is defined to be a sequence of fiscal (monetary) plays by each policymaker, where both policymakers have only the fiscal (monetary) instrument available in each period, determined prior to the initial period.[5]

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[5] If a policymaker were able to use two policy instruments simultaneously, with two targets, he

Two parameter sets will be used in the following analysis:[6]

	d_1	d_2	d_3	a_1	a_2	Ψ_1	Ψ_2
P.S.#1:	0.2	0.3	0.2	0.2	1.0	0.5	0.7
P.S.#2:	0.7	0.3	0.2	0.2	1.0	0.5	0.7

Identical parameter values hold for the foreign country via the symmetry assumption. As previously discussed, the only parametric asymmetry in the model exists for the respective country's "wage contracting profile" (γ and γ^*). Parameter set #1 (P.S.#1) differs from parameter set #2 (P.S.#2) only in the "degree of openness" parameter (d_1 and $d_1{}^*$). The magnitude of this parameter determines the extent to which a country's "wage contracting profile" is directly transmitted abroad. This can be seen in equations (4), where the domestic country's contribution is reproduced here for convenience:

$$p_{it} = (1-d_1)w_t + d_1(w_t{}^* + e_t)$$

As discussed in Section 2, a greater degree of openness (i.e., a larger d_1 value) means that the foreign country's wage behavior, as well as nominal exchange rate movements, directly impinge upon the domestic country's long run price level, and so, on its price level movements over time. This is important since, as can be seen below, a less insulated economy must, by virtue of the parametric structure of its economy, "eat" some of the other country's volatility without

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would be able to achieve his bliss point regardless of the other country's policy actions. Hence, the question of policy coordination would be moot.

[6] Parameter values are obtained from previous studies. In particular, see Currie and Levine [1985].

demanding some welfare–enhancing tradeoff.

In order to maintain a focus on the labor market asymmetry between the countries, only symmetric aggregate demand and labor shocks are considered. These shocks are assumed to be random disturbances which occur in the first period of the game. The qualitative game–theoretic results are similar for the aggregate demand and labor shocks, while the latter generally leads to greater quadratic losses in any particular game. For expositional simplicity, only symmetric aggregate demand shocks will be presented.[7]

A. The Welfare Effects of Differing "Wage Contracting Profiles"

A country's welfare is proxied by a quadratic loss function which measures the squared deviations of its output and price levels from their steady–state values (normalized to zero). Therefore, the question of the welfare effects of differing "wage contracting profiles" is, in essence, a question regarding the impact of these differing "profiles" on the volatility of output and price levels. That is, do shorter (less staggered) contracts lead to more or less volatility in output and price levels? The general result is that shorter contract lengths (i.e., larger values of $\gamma(\gamma^*)$),ceteris paribus, yield larger quadratic loss values for <u>all</u> countries. For example, assuming that the foreign country's contract lengths become shorter so that there is an increase in γ^* (with γ constant), there is a welfare loss in <u>both</u> the foreign and domestic countries. The increased volatility is shared by all participants. Consider the following fiscal and

[7] A "period" can most easily be thought of in the following discussion as a year. This designation will be used in what follows, and they will be used interchangeably. Also, the initial conditions of 'predetermined' state variables (y,y^*,p,p^*) are set equal to zero, while the 'jump' state variables are: $w_0 = w_0^* = 0.5$ and $e_0 = 0.0$. In addition, it is assumed that each target is given equal weight in each policymaker's optimization problem, i.e,. $\beta = \beta^* = 0.5$

monetary games under parameter set #1 in which each country is exposed to an aggregate demand shock of one unit in the first period of the game. A five period horizon is assumed for each policymaker.

1. Fiscal Game

The Fiscal policy game is illustrated in Figures 2 & 3.

Figure 2: Cooperative frontier for a five period Fiscal game (P.S.#1)

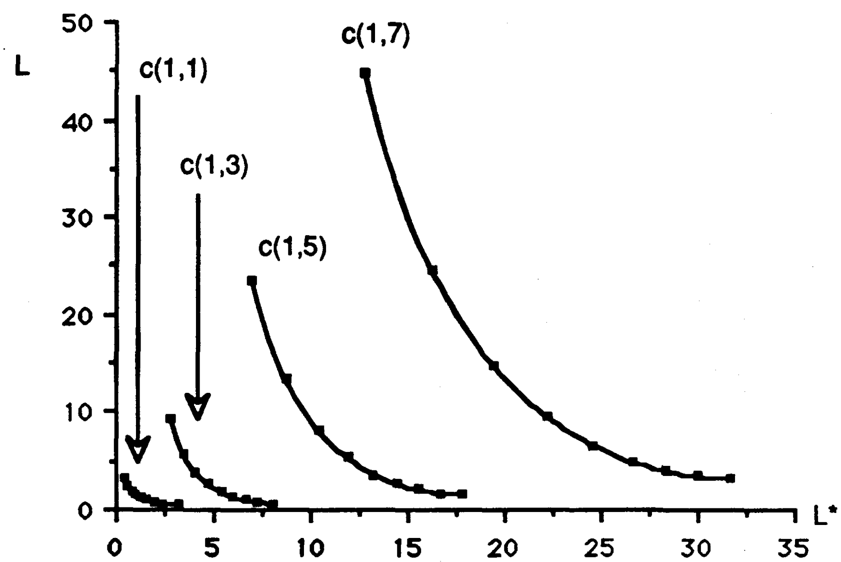


Figure 3: Nash "bargaining" solution & Nash "noncooperative" solution for a five period Fiscal game (P.S.#1)

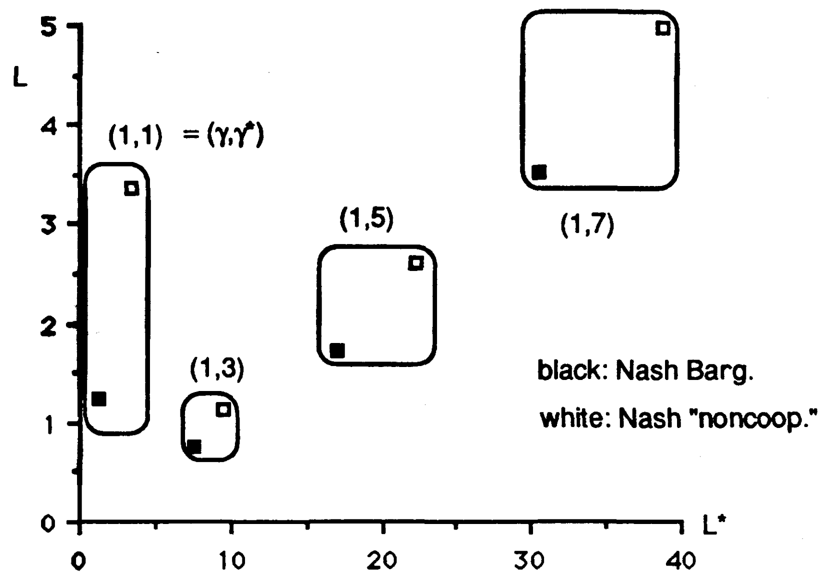


Figure 2 shows the outward shift in the cooperative frontier for increasing values of γ^* (with γ=1), i.e., for shorter foreign contract lengths. Only one point can be chosen from the cooperative frontier to represent the cooperative equilibrium point. As discussed above, many different cooperative equilibria exist all with the proviso that it be Pareto superior to some "threat" point. The cooperative equilibrium point chosen here is the Nash "bargaining" solution with the "threat" point being the Nash "noncooperative" equilibrium. These points are shown in Figure 3 for increasing values of γ^*, with the Nash "bargaining" solution and the Nash "noncooperative" solution designated by the black and white characters, respectively. As discussed above, the general trend is for larger quadratic loss values (lower welfare) in both countries as the foreign contract length decreases. The greater price and output volatility for shorter foreign contract lengths in the fiscal game is shown in Figures 4 and 5.

Figure 4: Price level volatility for a five period Nash "noncooperative" Fiscal game (P.S.#1)

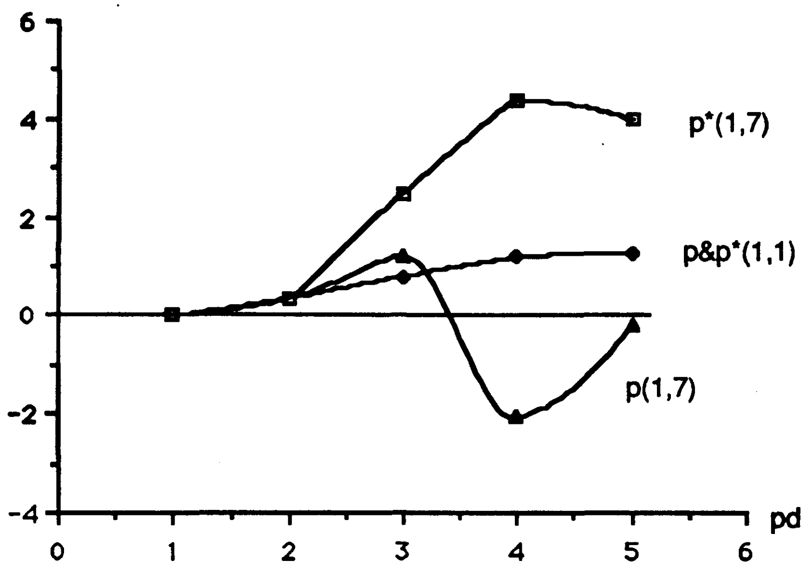
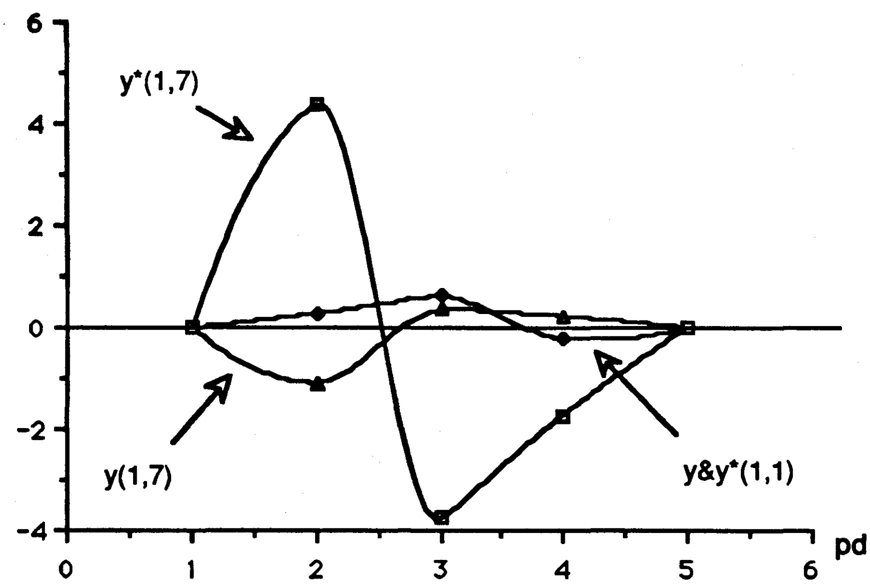


Figure 5: Output volatility for a five period Nash "noncooperative" Fiscal game (P.S.#1)



Figures 4 and 5 show the evolution of the price level and output, respectively, over the five periods of the "noncooperative" Nash equilibria for symmetric ($\gamma=\gamma^*=1$) and asymmetric ($\gamma=1,\gamma^*=7$) "wage contracting profiles". These pictures confirm the result observed in Figure 3. Greater price and output volatility in both countries follow a shortening of foreign contracts. In the Fiscal game, greater volatility accrues to the country which experiences the shorter contract length (i.e., to the foreign country). This is also supported by Figure 3 in which the foreign country experiences greater quadratic losses (in absolute terms) relative to the domestic country.

Table 1[a]

	$L_O = L^*_O = 0.2586$ (7.7%)	
	$\uparrow$	
$(\gamma, \gamma^*) = (1,1)$	$L = L^* = 3.3631$	
	$\downarrow$	
	$L_P = L^*_P = 3.1045$ (92.3%)	
	$L_O = 0.6795$ (13.6%)	$L^*_O = 18.0896$ (46.6%)
	$\uparrow$	$\uparrow$
$(\gamma, \gamma^*) = (1,7)$	$L = 4.9825$	$L^* = 38.7984$
	$\downarrow$	$\downarrow$
	$L_P = 4.3030$ (86.4%)	$L^*_P = 20.7088$ (53.4%)

a: A 5 period Nash "noncooperative" Fiscal game is played, with a joint aggregate demand shock of one unit in the initial period. L (L^*) is the domestic (foreign)quadratic loss value, L_O (L^*_O) and L_P (L^*_P) are the domestic (foreign) quadratic loss components attributed to ouput volatility and price volatility, respectively. The percentage which each component contributes to the total quadratic loss is denoted parenthetically.

Table 1 presents the quadratic loss outcomes for a 5 period Nash "noncooperative" Fiscal game. The symmetric case — $(\gamma, \gamma^*) = (1,1)$ — yields identical quadratic losses for the domestic and foreign countries, i.e., $\frac{L}{L^*} = 1.00$. 92.3% of the total quadratic loww in each country stems from price volatility, while the remaining 7.7% is caused by output deviations from steady–state over the five periods of the Fiscal game. Turning to the asymmetric case — $(\gamma, \gamma^*) = (1, 7)$ — the foreign quadratic loss is 7.7869 times greater than that of the domestic country, i.e., $\frac{L}{L^*} = 7.7869$. For the domestic country, most of the loss is still caused by price volatility where it comprises 86.4% of the total loss as compared to only 13.6% due to output volatility. For the foreign country, a larger portion of the total quadratic loss is due to output volatility when compared to the symmetric case. 46.6% of the total foreign loss is due to output deviations from steady–state while 53.4% is due to price volatility.

Figures 6 and 7 show the evolution of the "jump" variables (w, w*, and e) and the fiscal policy plays for the 5 period Nash "noncooperative" game. From Figure 6 one can see that nominal wage rates in both countries are relatively stable for identical "wage contracting profiles" $(\gamma = \gamma^* = 1)$, and the nominal exchange rate is constant at zero since this reduces to the perfectly symmetric case and there exists no wedge to drive the exchange rate away from its steady–state level. In addition, the fiscal policy plays are identical for each country in the perfectly symmetric case as expected (see Figure 7). Introducing a shorter foreign "wage contracting profile" — $(\gamma, \gamma^*) = (1,7)$ — one finds that the domestic and foreign fiscal plays are quite distinct. In fact, the fiscal plays tend to be opposite one another. When the domestic country enacts a contractionary fiscal policy the foreign country enacts an expansionary stance and vice versa. The greater upward foreign price level movement causes a foreign monetary disequilibrium which necessitates an upward movement in the foreign interest rate above that in the domestic country (an effect which is supported by the downward movement in the domestic price level and constant money supplies). Hence, the nominal exchange rate appreciates (with respect to the domestic country).

Figure 6: Nominal Wage Rates and Exchange Rate for a five period Nash "noncoopertive" Fiscal Game (P.S. #1)

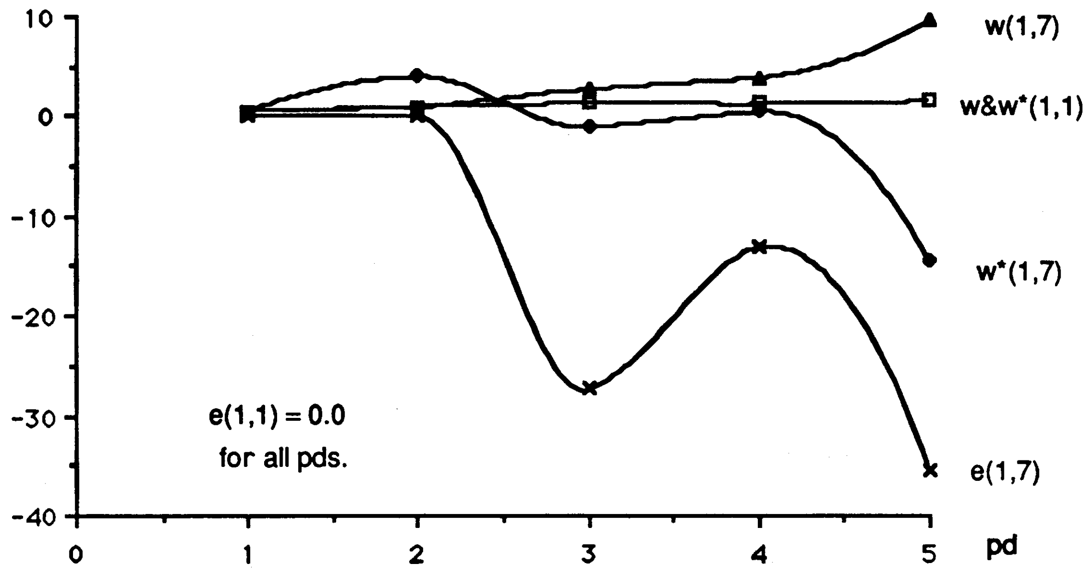
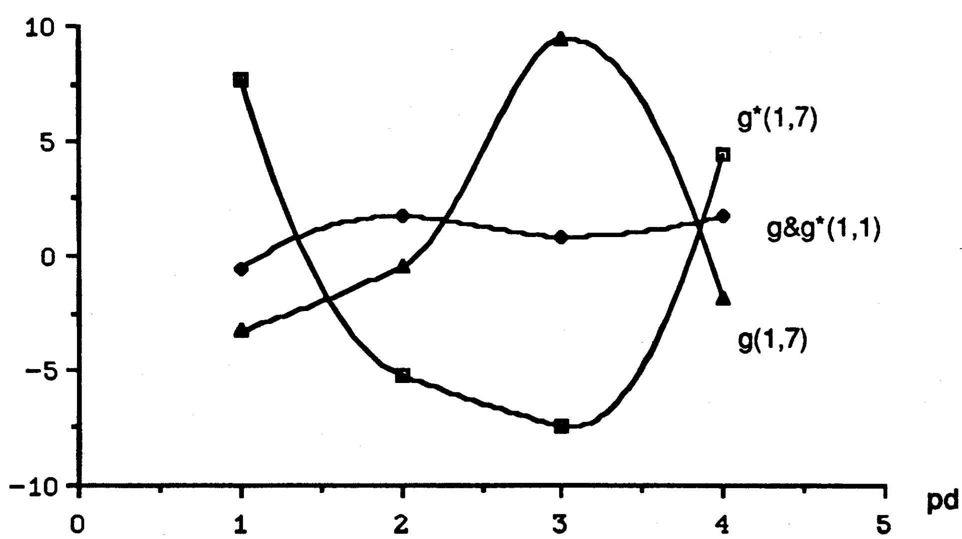


Figure 7: Fiscal Policy for a five period Nash "noncooperative" Fiscal Game (P.S. #1)



2. Monetary Game

Consider now the Monetary game. The outward shift of the cooperative frontier and the movement of the Nash "bargaining" – "noncooperative" solutions for shorter foreign contract lengths are shown in Figures 8 and 9, respectively.

Figure 8: Cooperative frontier for a five period Monetary game (P.S.#1)

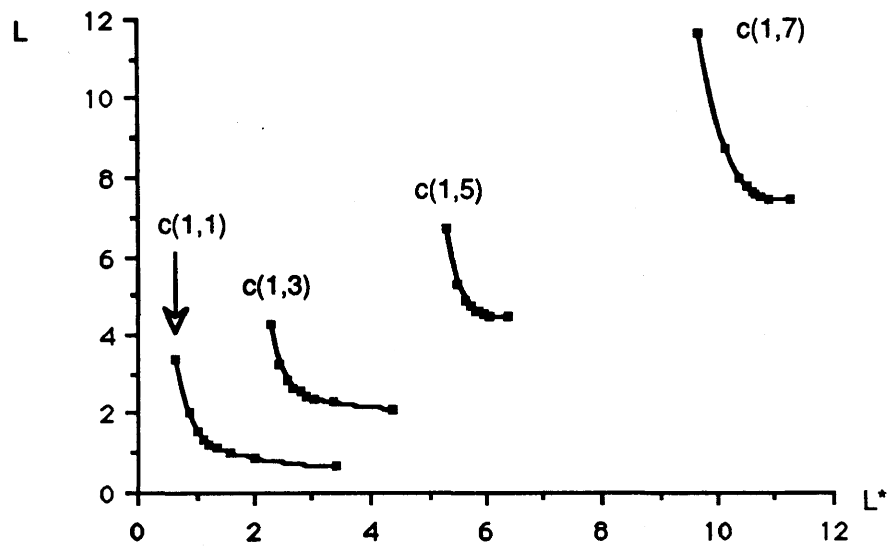
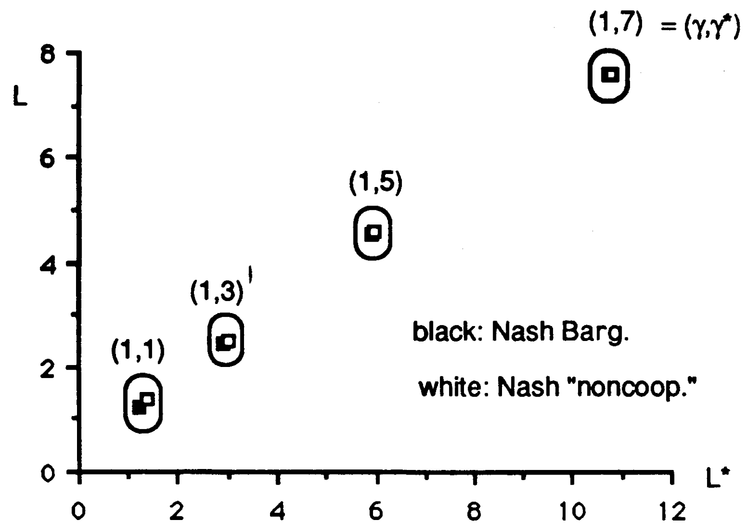


Figure 9: Nash "bargaining" solution & Nash "noncooperative" solution for a five period Monetary game (P.S.#1)



Figures 8 and 9 of the Monetary game are the analogs of Figures 2 and 3 of the Fiscal game. Again, in Figure 9 the Nash "bargaining" solution and the Nash "noncooperative" solution are identified by the black and white characters, respectively. As in the Fiscal game, quadratic losses are larger for shorter foreign contract lengths in the Monetary game. Casual observation of Figures 8 and 9 show that the quadratic losses are less favorably skewed toward the domestic country in the Monetary game as compared to the Fiscal game for the five period horizon. In addition, there is very little difference between the "bargaining" and "noncooperative" equilibria in the Monetary game. The evolution of output and price levels for the Nash "noncooperative" Monetary game are illustrated in Figures 10 and 11.

Figure 10: Price level volatility for a five period Nash "noncooperative" Monetary game (P.S.#1)

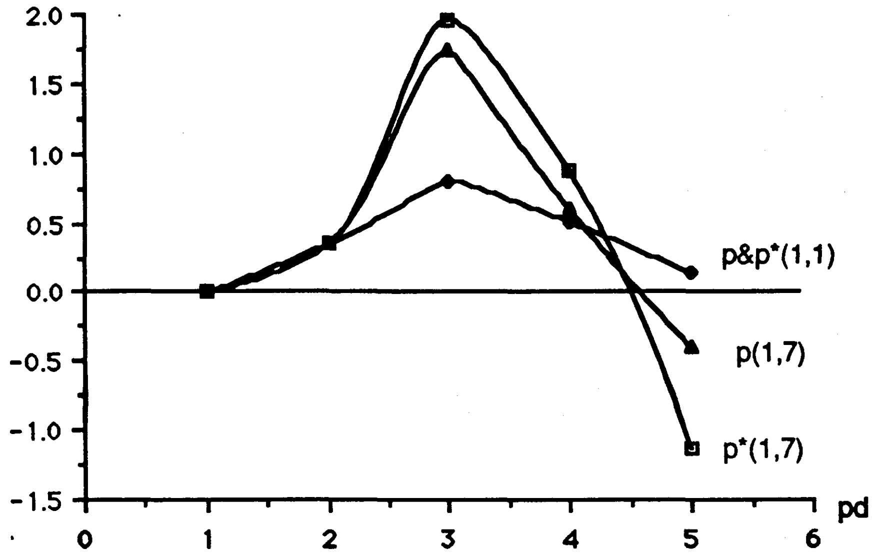


Figure 11: Output volatility for a five period Nash "noncooperative" Monetary game (P.S.#1)

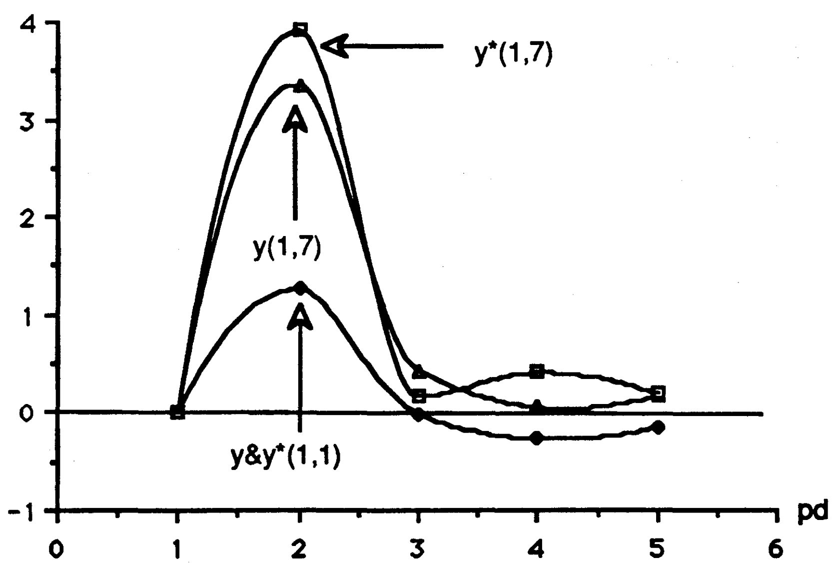


Figure 11: Output volatility for a five period Nash "noncooperative" Monetary game (P.S.#1)

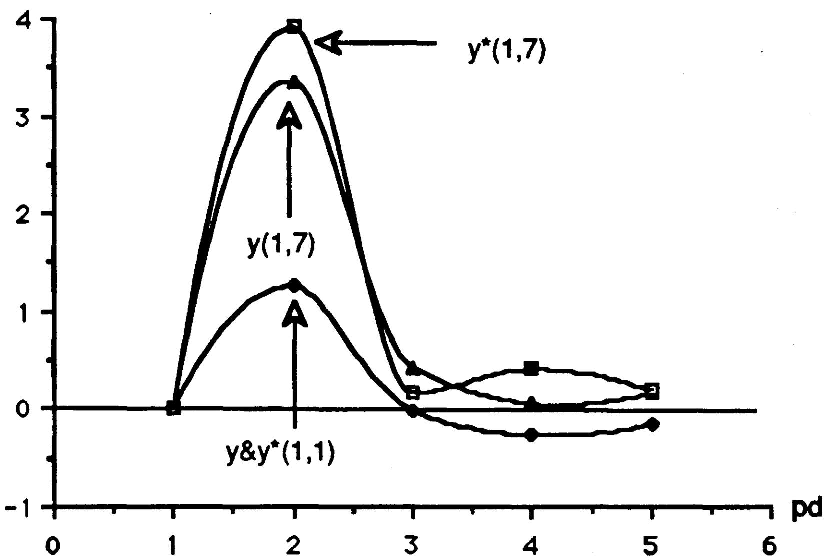


Table 2[a]

$(\gamma, \gamma^*) = (1,1)$

$L_O = L^*_O = 0.8482$ (61.2%)

↑

$L = L^* = 1.3866$

↓

$L_P = L^*_P = 0.5384$ (38.8%)

$(\gamma, \gamma^*) = (1,7)$

Domestic	Foreign
$L_O = 5.7270$ (75.5%)	$L^*_O = 7.7398$ (71.8%)
↑	↑
$L = 7.5844$	$L^* = 10.7801$
↓	↓
$L_P = 1.8574$ (24.5%)	$L^*_P = 3.0403$ (28.2%)

a: A 5 period Nash "noncooperative" Monetary game is played, with a joint aggregate demand shock of one unit in the initial period. L (L*) is the domestic (foreign)quadratic loss value, L_O (L^*_O) and L_P (L^*_P) are the domestic (foreign) quadratic loss components attributed to ouput volatility and price volatility, respectively. The percentage which each component contributes to the total quadratic loss is denoted parenthetically.

Table 2 presents the quadratic loss outcomes for a 5 period Nash "noncooperative" Monetary game. As in the Fiscal game, the symmetric case — $(\gamma, \gamma^*) = (1, 1)$ — yields identical quadratic loss values for the domestic and foreign countries, i.e., $\frac{L}{L^*} = 1.00$. 61.2% of the total loss is due to output volatility while the remaining 38.8% is due to price volatility in each country. Turning to the asymmetric case — $(\gamma, \gamma^*) = (1, 7)$ — output volatility plays a primary role in the total quadratic losses for both countries. The foreign quadratic loss is 1.4214 times greater than the domestic quadratic loss, i.e., $\frac{L}{L^*} = 1.4214$. 75.5% of the total domestic quadratic loss is caused by output deviations from steady–state, with 24.5% caused by price volatility. For the foreign country, 71.8% of the quadratic loss is due to output volatility and 28.2% due to price volatility.

Figures 12 and 13 show the evolution of the "jump" variables (w, w*, and e) and the monetary policy plays for the 5 period Nash "noncooperative" game. Again, the domestic and foreign policymakers form policy identically and the endogenous variables evolve identically in the symmetric case — $(\gamma, \gamma^*) = (1,1)$. Figures 10 and 13 can be combined to show that the foreign real money supply expands by more than does the domestic real money supply in every period with a less than offsetting foreign price level expansion in the first four periods of the game. The fall in the foreign price level in the final period can be explained by the large exchange rate appreciation in terms of the foreign country. The foreign excess real money supply necessitates a decrease in the foreign interest rate below that of the domestic rate, and so, a nominal exchange rate depreciation. Again, as shown in Figure13, the policy stances of the domestic and foreign policymakers differ as the foreign country takes a more expansive monetary stance than does the domestic country in every period of the game.

Figure 12: Nominal Wages and Exchange Rate for a five period Nash "noncooperative" Monetary Game (P.S. #1)

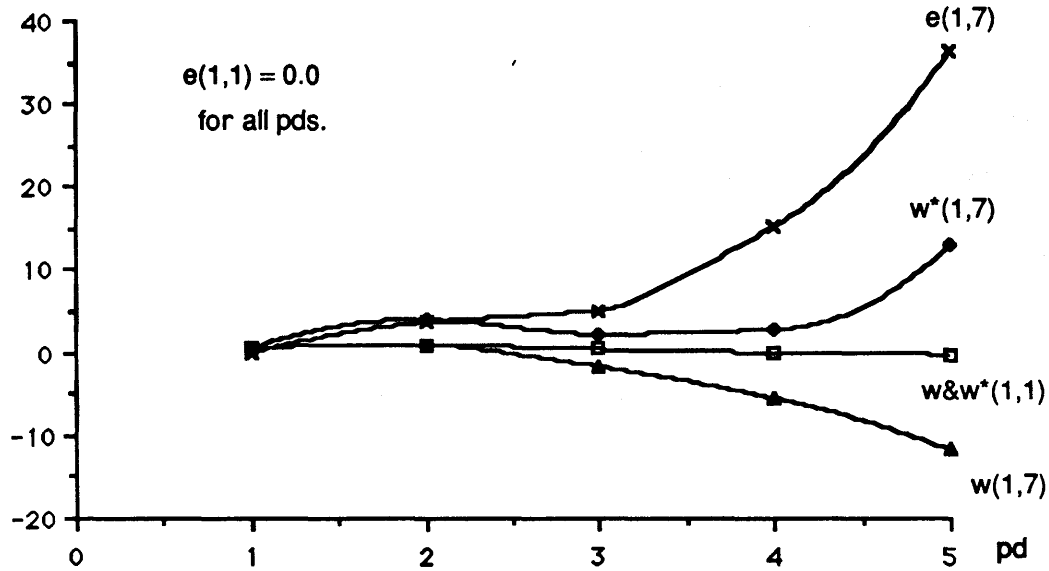
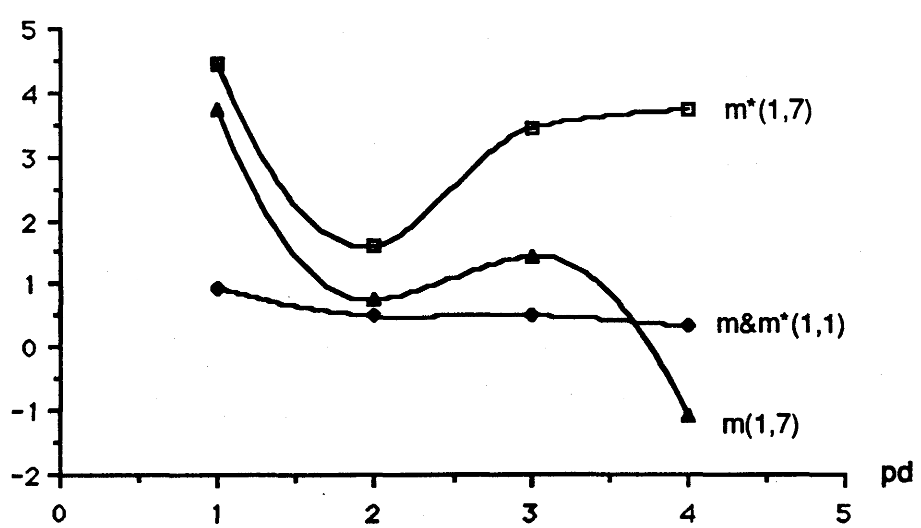


Figure 13: Monetary Policy for a five period Nash "noncooperative" Monetary Game (P.S. #1)



The Monetary game results are similar to those observed in the Fiscal game. Increased volatility in both countries result from a shortening of contract lengths in only one of the countries. Regardless of the equilibrium concept and the policy game, the simulation exercises yield the comparative-static result:

$$\frac{\partial L}{\partial \gamma^*} > 0 \; ; \; \frac{\partial L^*}{\partial \gamma^*} > 0.$$

B. The Effect of the Policymakers' "Time Horizon"

It is generally accepted that the time horizon of policymakers is fundamentally conditioned by the particular political institutional framework. This section considers how the length of the time horizon affects for the evolution and ultimate outcome of the dynamic policy games. In general, longer time horizons yield outcomes which are more advantageous to the country with the shorter contract lengths. Policymakers in the short contract country are better able to minimize price and output volatility when policy games are played over a longer period of time. As in the previous section, an aggregate demand shock of one unit is assumed to strike both countries in the first period. Parameter set #1 will again be used in the following analysis. A five year horizon (short horizon) will be contrasted to a ten year horizon (long horizon).

1. Fiscal Game

Consider the Fiscal game under the short and long horizons. The Nash "bargaining" and "noncooperative" equilibria are shown for "wage contracting profile" couplet: $(\gamma,\gamma^*) = (1,7)$.

Figure 14: Nash "bargaining" & "noncooperative" equilibria for a five period Fiscal game (P.S.#1)

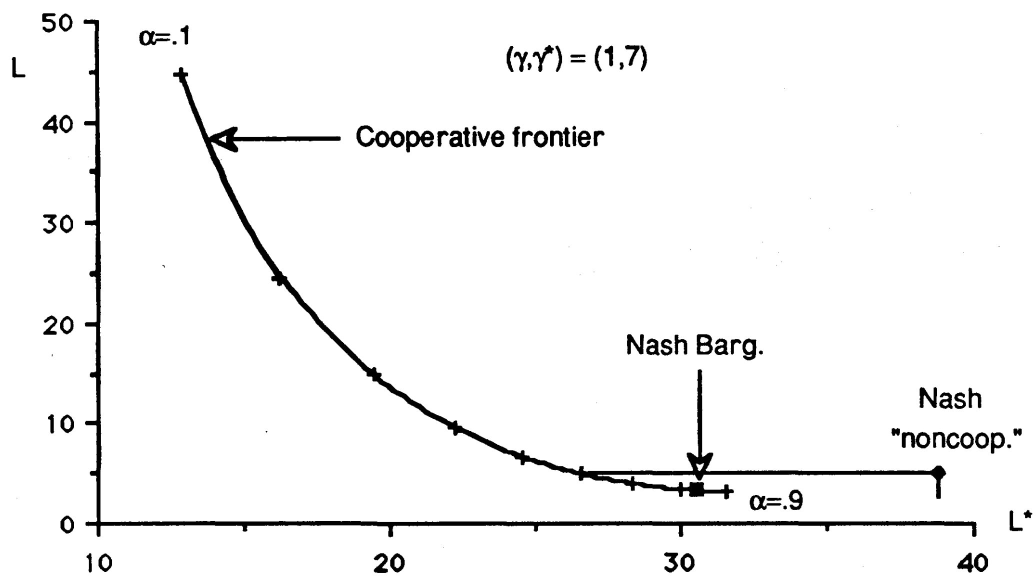
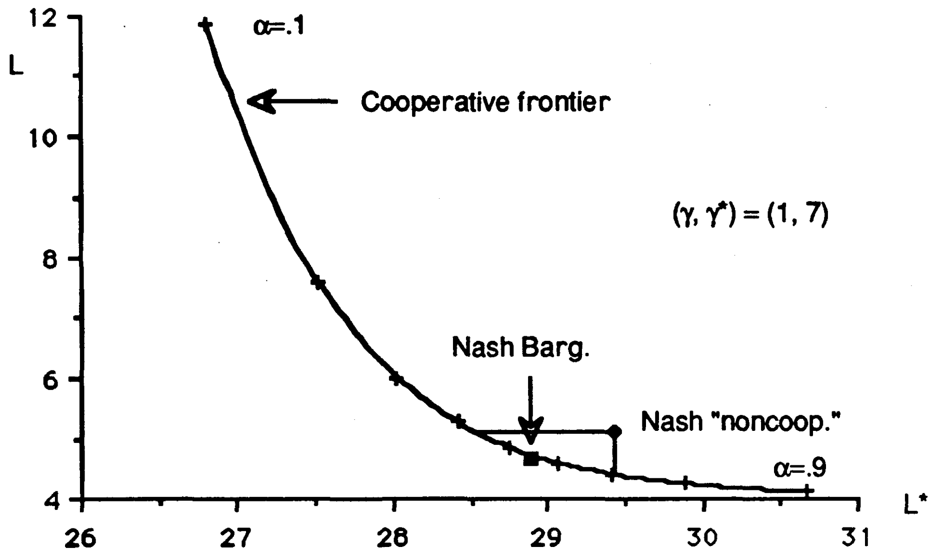


Figure 15: Nash "bargaining" & "noncooperative" equilibria for a ten period Fiscal game (P.S.#1)



The claim being made here is that, under the Fiscal game, the short contract length country (foreign country) fairs better under the long horizon. Figures 14 and 15 demonstrate this well. Although the foreign country's quadratic losses are much greater than that of the domestic country under both time horizons, the terms under which a cooperative agreement may be struck are far more advantageous for the foreign country in the long horizon game as compared to the short horizon.

The *negotiation set*[8] can be used as a measure of a country's bargaining power. In the five period horizon game (Figure 14) the negotiation set extends from α=.6 to $\alpha > .9$.[9] The Nash "bargaining" equilibrium is (L,L*) = (3.5095,

---

[8] The negotiation set is defined as those points along the cooperative frontier which are Pareto superior to the "threat point" (i.e., the Nash "noncooperative" equilibrium).

[9] Recall that α represents the proportion of the domestic policy goal which is taken into account in the cooperative agreement, and so, $(1-\alpha)$ is the weight given the foreign policy goal. Therefore, larger values of α denote more favorable terms for the domestic country under the cooperative agreement, and vice versa.

30.5709) with α=.836. In contrast, for the ten period horizon game (Figure 15), the boundaries of the negotiation set are $.43 < \alpha < .7$. The "bargaining" equilibrium is $(L,L^*) = (4.6636, 28.8918)$ with α=.54. Clearly, the foreign country is in a much better bargaining position under the ten period game where it can forge a more equitable cooperative equilibrium. This is supported by the result that the losses associated with both the Nash "bargaining" and "noncooperative" equilibria are lower in absolute terms for the ten period game eventhough it includes the five additional periods. The implication is that the short contract country is better able to curb its price and output volatility when policy is formed over a longer horizon. Figures 16 and 17 illustrate the evolution of price and/or output for the five and ten period Fiscal game in the domestic and foreign countries, respectively, under the Nash "noncooperative" rules.

Figure 16: Evolution of domestic price and output for a five & ten period Nash "noncooperative" Fiscal game (P.S.#1)

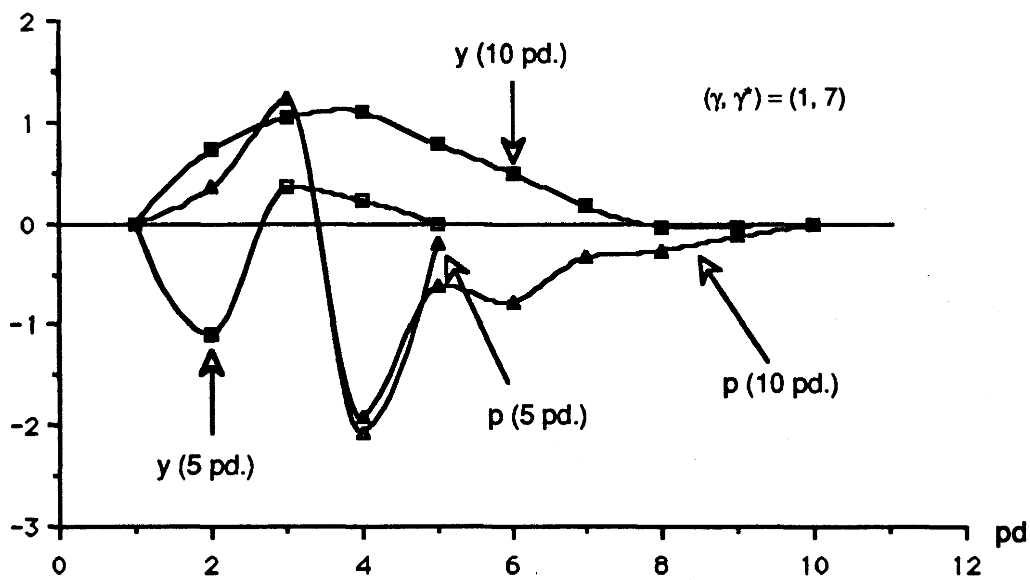
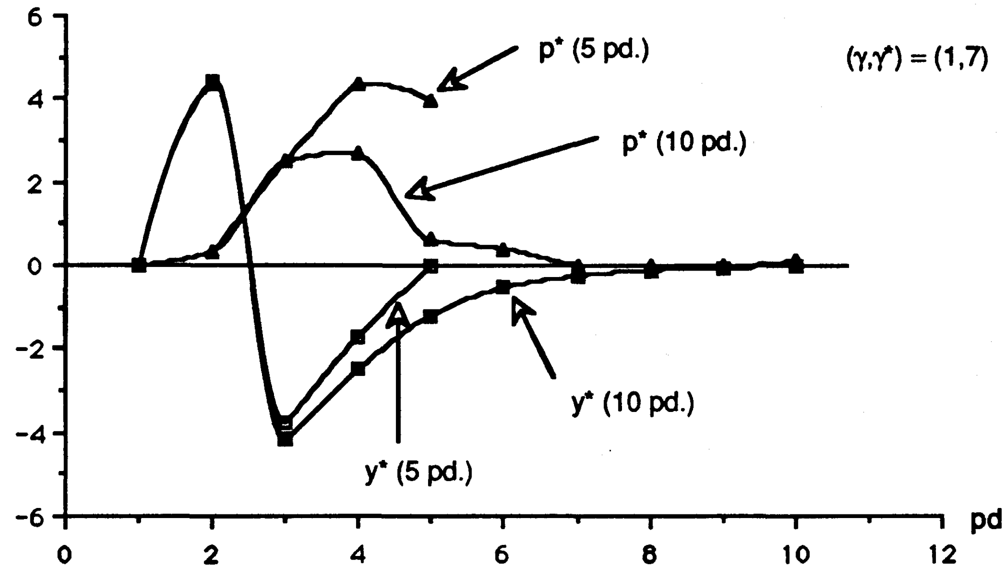


Figure 17: Evolution of foreign price & output for a five & ten period Nash "noncooperative" Fiscal game (P.S.#1)



As Figures 14 and 15 demonstrate improved welfare outcomes are obtained by the short contract country (foreign country) in the long horizon game. This beneficial welfare effect is primarily due, as shown by Figure 17, a moderation in price level increases for the ten period game. Foreign output volatility is comparable for both horizon games.

2. Monetary Game

Now consider the Monetary game under a joint aggregate demand shock in the initial period. The five and ten period games are illustrated in Figures 18 and 19.

Figure 18: Nash "bargaining" & "noncooperative" equilibria for a five period Monetary game (P.S.#1)

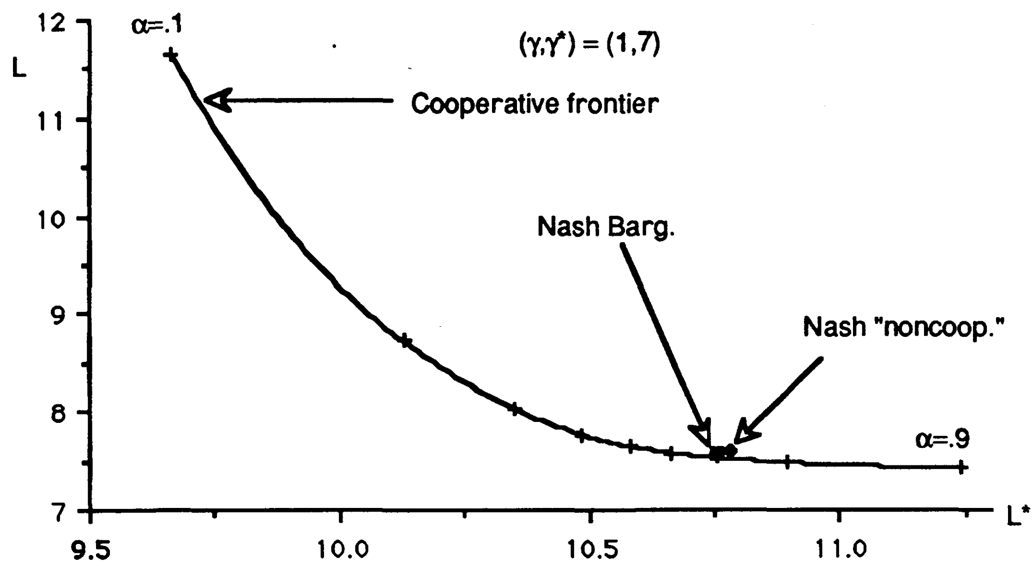
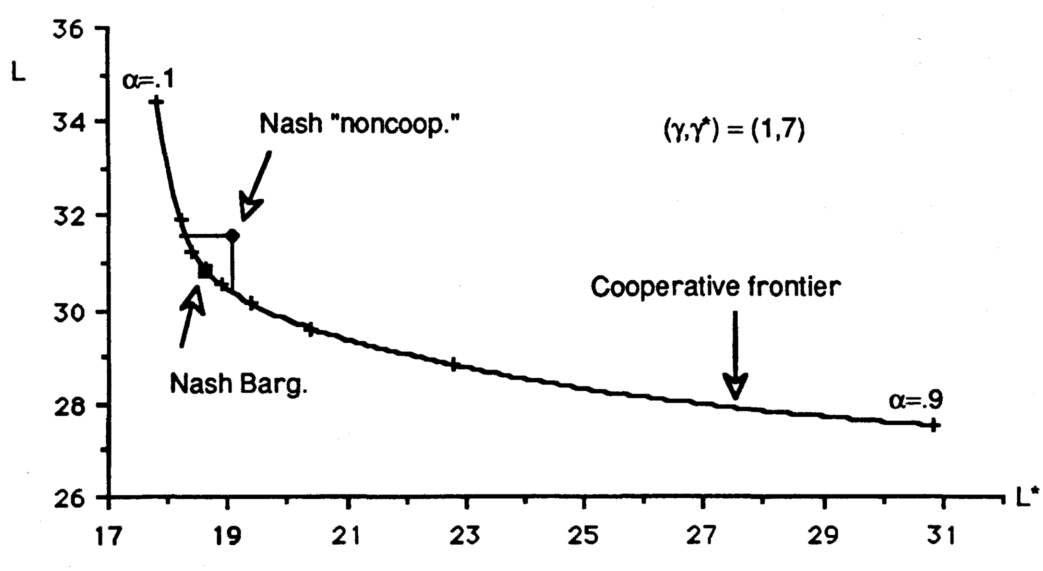


Figure 19: Nash "bargaining" & "noncooperative" equilibria for a ten period Monetary game (P.S.#1)



Again, the short contract length country (foreign country) is better off relative to the domestic country in the ten period Monetary game as compared to the five period game. Under the five period horizon, the "bargaining" equilibrium is $(L,L^*) = (7.5719, 10.7539)$ — $\alpha=.7$ — with the negotiation set being $.59 < \alpha < .72$. The ten period horizon game yields a "bargaining" equilibrium of $(L,L^*) = (30.8075, 18.6247)$ — $\alpha=.42$ — with the negotiation set being $.25 < \alpha < .53$. Not only is the bargaining position of the short contract country improved in the ten period horizon, the welfare level of the short contract country is greater than that of the long contract country (i.e., the quadratic losses are smaller). These cooperative welfare results are mirrored by the "noncooperative" equilibria for the short and long time horizons. The Nash "noncooperative" equilibria can be decomposed into each country's price and output volatility for the five and ten period horizons.

Figure 20: Evolution of domestic price and output in a five and ten period Nash "noncooperative" Monetary game (P.S.#1)

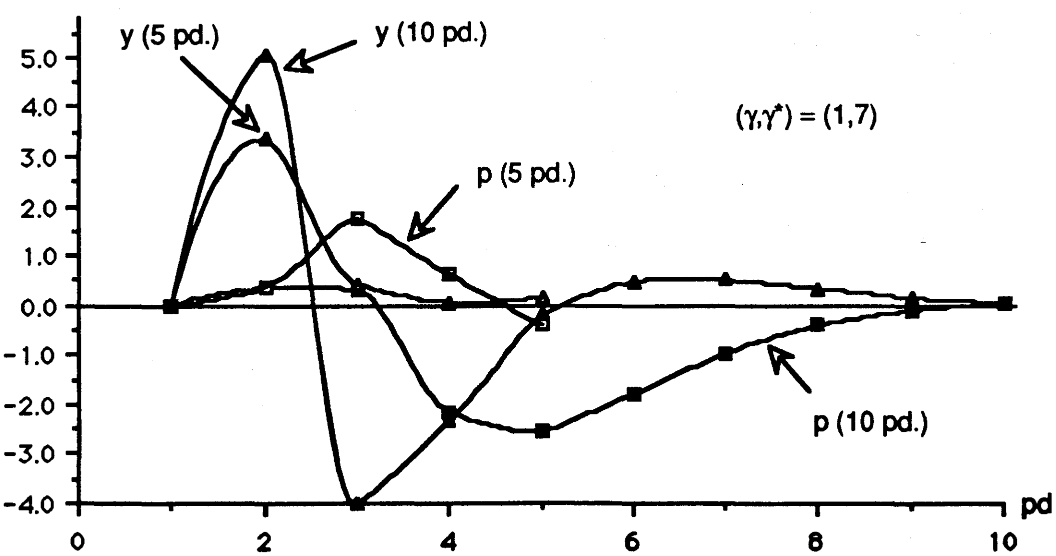
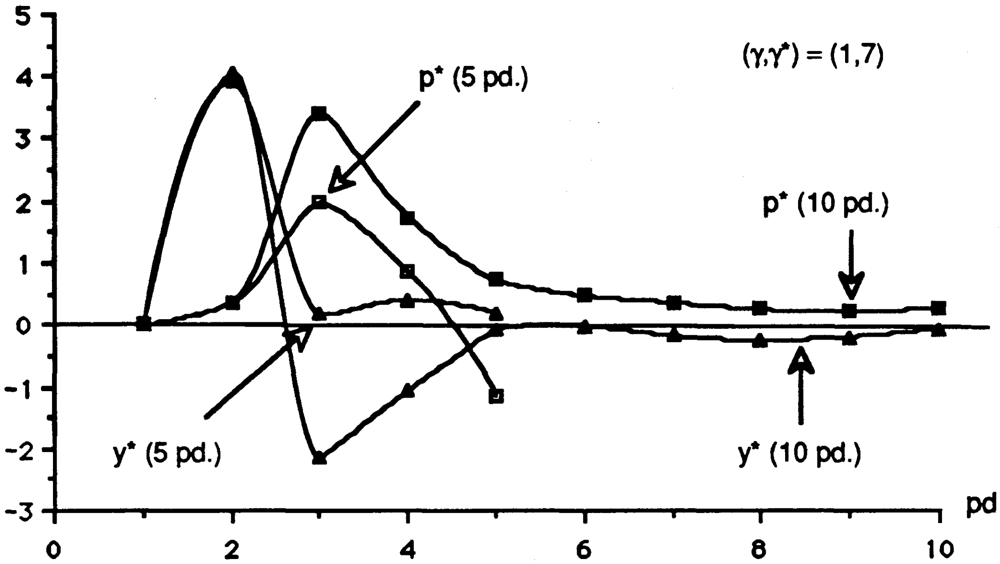


Figure 21: Evolution of foreign price and output for a five and ten period Nash "noncooperative" Monetary game (P.S.#1)



The effectiveness of monetary and fiscal policies in this setting can be gauged by the average welfare per period for both horizons. In this way, quadratic loss values may be placed on equal footing.

Table 3[a]

	Quadratic Loss[b]		Average Welfare Loss[b]		Negotiation Set	
	L	L^*	$\frac{L}{pd}$	$\frac{L^*}{pd}$	α_L	α_R
Fiscal						
5 pd:	4.98	38.80	.996	7.76	.6	.9+
10 pd:	5.11	29.44	.511	2.94	.43	.7
Monetary						
5 pd:	7.58	10.78	1.52	2.16	.59	.72
10 pd:	31.56	19.08	3.16	1.91	.25	.53

a:A joint aggregate demand shock of one unit in the initial period is assumed along with the "wage contracting profile" couplet $(\gamma,\gamma^*)=(1,7)$;

b:These are the Nash "noncooperative" equilibrium values.

Table 3 neatly summarizes the previous discussion concerning the welfare implications of the chosen time horizon. The short contract length country (foreign country), for both the Monetary and Fiscal games, achieves a lower 'per period' volatility level as well as an improved bargaining position with the ten period horizon as compared to the five period horizon. The improved bargaining position of the foreign country can come only at the expense of the long contract length (domestic) country. The domestic country achieves a lower average volatility level with the ten period horizon only in the Fiscal game.

C. Incentives to Form Cooperative Policy

The incentives and disincentives which individual countries have to form policy cooperatively are many and varied. They stretch beyond this study, in particular, and beyond economic analysis, in general. Rather, a complete answer to the study of policy cooperation must encompass the complex socio–economic–political interrelationships which define the country (or countries) in question. This study attempts to add to the broader task by analyzing the incentives and disincentives to form policy cooperatively for two countries which differ only in their "wage contracting profiles, that is, in the degree of staggering and length of their labor contracts.

Discussions on the incentives to cooperate focus on the welfare gains made by each country in forming a cooperative agreement (Nash "bargaining" equilibrium point) relative to the assumed "threat point" (Nash "noncooperative" equilibrium point), and by the weight given a country's policy goals in the cooperative agreement (given by α for the domestic country and $(1-\alpha)$ for the foreign country). The former measure is self–explanatory, while the latter measure can be understood by noting that there do exist limits to which any nation would sacrifice its own policy goals to form some cooperative agreement. For two countries which are identical in all ways except for this labor market asymmetry, there is no ostensible reason why either policymaker would settle for anything less than an equal share in the cooperative objective. Welfare gains under a cooperative arrangement must be sufficient to compensate a country for accepting less than an equal share.

Assuming that the world is composed of a "short horizon" policymaker, Table 4 presents the "Gains from Bargaining" and "Policy Objective Weights" for

Table 4 indicates that foreign welfare gain increases in the Fiscal game as the foreign wage contracts become shorter, while the domestic welfare gain decreases. For $(\gamma,\gamma^*) = (1,7)$, the foreign country has a welfare gain that is 5.59 times larger than the domestic country under P.S.#1 and 3.05 times larger under P.S.#2. Not only are the relative welfare gains greater for the short contract country under the Fiscal game with a low degree of "openness" (P.S.#1), they are far greater in absolute terms. Although having greater "Gains from Bargaining" in both Fiscal games, it comes at the expense of a much lower weight in the cooperative objective. In fact, the larger absolute gains in P.S.#1 relative to P.S.#2 require a greater sacrifice in the objective.

For the Monetary games, the "Gains from Bargaining" decrease in general as the foreign country's contract shortens. This result sharply differs from the Fiscal games. With a low degree of "openness" ($d_1 = d_1^* = 0.2$), the foreign country faces the undesirable prospect of smaller cooperative gains as its contract lengths shorten while also sacrificing more of its policy goals in the cooperative objective. For $(\gamma,\gamma^*) = (1,7)$, the short contract country achieves bargaining gains that are 2.08 times the long contract country. This is less than half of the relative gains made in the corresponding Fiscal game (Fiscal/P.S.#1). The Monetary game with a high degree of "openness" (Monetary/P.S.#2) is the only case for which $\alpha < 0.5$ and where the long contract country has greater bargaining gains.

What conclusions can be drawn from the above results regarding the incentives and disincentives to coordinate policy? Under the structural environment in which each country is relatively insulated from the other's wage contracting process (P.S.#1), coordination is an increasingly welfare–enhancing option for the short contract country in the Fiscal game if the short contract country is willing to pay the long contract country to absorb some of its additional volatility. The form of this remuneration is the sacrifice of a portion of its policy objective in the cooperative agreement. This cost is seemingly large in that the foreign policy goals receive a weight of only 0.16 for $(\gamma,\gamma^*) = (1,7)$. By way of contrast, the corresponding Monetary game yields Nash "bargaining" outcomes which are of decreasing benefit to each player while still requiring the short contract country to sacrifice some of its policy goals. For $(\gamma,\gamma^*) = (1,7)$, the cooperative weight of the foreign policy goals is 0.30.

With a larger degree of exposure to the wage contracting process abroad (P.S.#2) the gains from cooperation are more equally shared among the competing countries and the cooperative weights are more equally distributed as compared to their P.S.#1 counterparts. The effect of the degree of "openness" on the five period game–theoretic outcomes is illustrated in Figures 22 and 23.

Figure 22: Degree of "Openness" for a five period Fiscal game

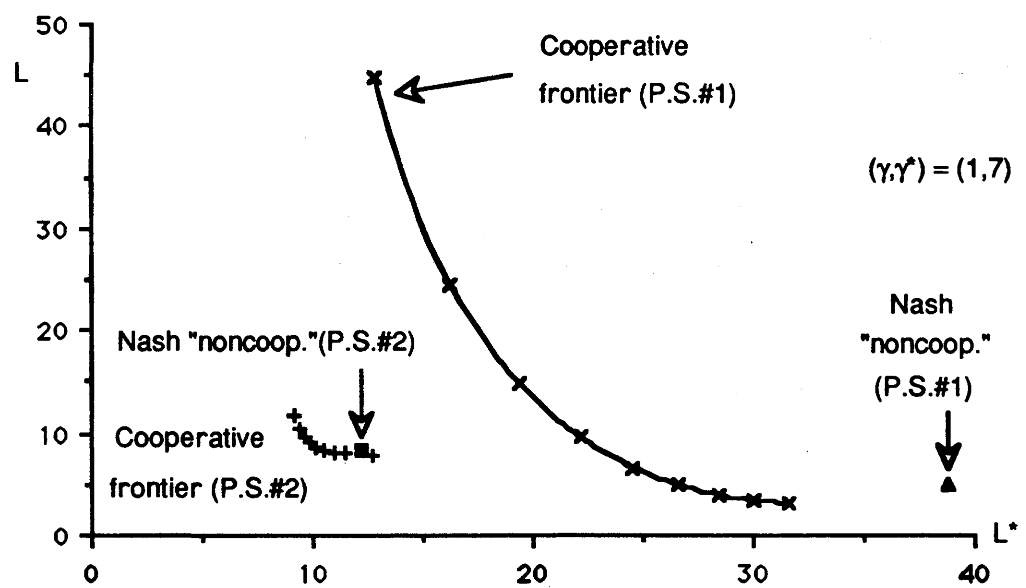
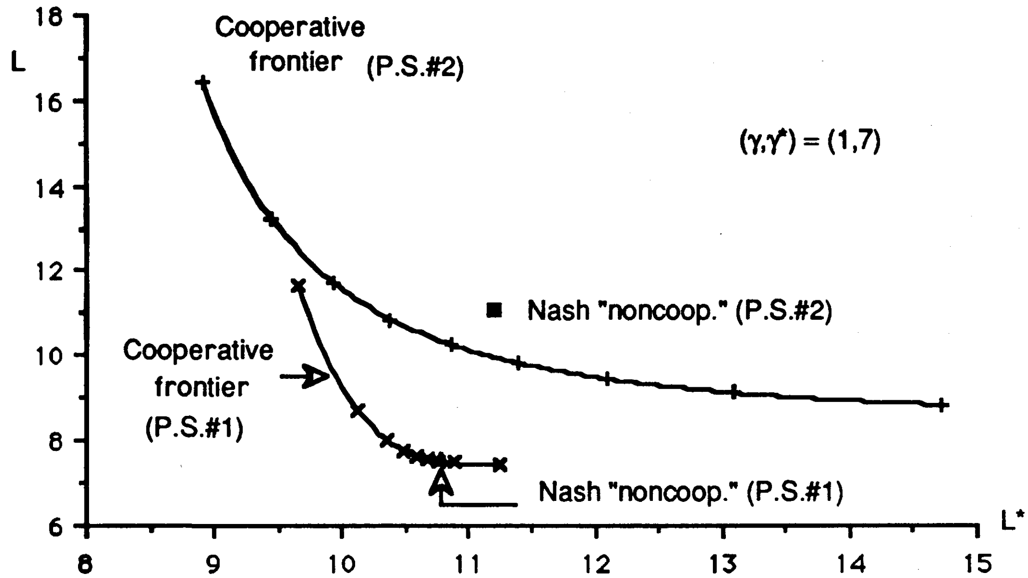


Figure 23: Degree of "Openness" for a five period Monetary game



Quadratic loss values are lower in the Fiscal game (Figure 22) for the short contract length country when wage contracting exposure is high. Losses are higher for the long contract length country. The foreign country is able to export some of its volatility to the domestic country by virtue of the parametric structure of the economy. In contrast to this is the Monetary game (Figure 23) in which both countries suffer higher quadratic loss values for larger degrees of "openness". In this case the long contract country is the prime beneficiary of reduced wage contract exposure from abroad.

Some general results are worth noting. The long contract length country is never better off by having increased vulnerability to the wage contracting process of the short contract length country. In addition, quadratic losses are more equally shared under both Monetary and Fiscal games when there is a high degree of "openness". The bargaining position of the foreign country (represented by $1-\alpha$) also improves when mutual exposure is high. The intuition here is that the parametric structure of the economy forces, when the degree of "openness" is large, the domestic country to absorb volatility from abroad without demanding compensation.

D. Dominant Strategies [10]

Thus far, only matched policy plays, in which the policy instruments are identical in the two countries, have been used in analyzing the welfare effects of differing "wage contracting profiles". This section addresses the question of whether "dominant strategies" (i.e., policy plays) exist for particular players (short or long contract length countries). Nash "noncooperative" equilibria for a symmetric aggregate demand shock of one unit in the initial period are compared to determine whether particular policy instruments yield more desireable game–theoretic welfare outcomes. A dominant strategy is defined as follows. Denote N_{ij} as the domestic quadratic loss where the domestic country plays policy i (i=M,F) given that the foreign country plays policy j (j=M*,F*). Likewise, N^*_{ij} represents the foreign quadratic loss where the foreign country plays policy i (i=M*,F*) in response to a domestic policy play of j (j=M,F). Fiscal policy is a "dominant strategy" for the domestic country if:

$$N_{FM^*} < N_{MM^*} \; ; N_{FF^*} < N_{MF^*}.$$

[10] A dominant strategy in this context refers to games in which policymakers can choose only one instrument at the beginning of the game and must use this instrument for all periods of the game. This is contrasted with the case in which policymakers have only one instrument at their disposal but may choose the instrument in every period.

Simply put, the domestic country is <u>always</u> better off by using fiscal policy, irrespective of the policy choice by the foreign country. Similarly, foreign fiscal policy is a "dominant strategy" if:

$$N^*_{F^*M} < N^*_{M^*M} \; ; \; N^*_{F^*F} < N^*_{M^*F}.$$

The converse holds for monetary policy in both countries. The quadratic loss outcomes are presented in Table 5.

Table 5[a]

(γ,γ^*) /	N_{MM^*}	N_{FM^*} /	N_{MF^*}	N_{FF^*} /	$N^*_{M^*M}$	$N^*_{F^*M}$ /	$N^*_{M^*F}$	$N^*_{F^*F}$ /
5 pd/P.S.#1								
(1,1) /	1.39	2.71 /	1.50	3.36 /	1.39	2.71 /	1.50	3.36 /
(1,3)	2.52	160.40	2.12	1.14	3.00	4.67	6.47	9.53
(1,5)	4.56	1599.84	3.13	2.62	5.98	9.79	859.84	22.20
(1,7)	7.58	537.72	4.42	4.98	10.78	16.46	777.33	38.80
5 pd/P.S.#2								
(1,1) /	2.17	2.20 /	2.56	1.29 /	2.17	2.20 /	2.56	1.29 /
(1,3)	3.70	318.96	4.36	2.61	3.24	3.41	94.28	3.13
(1,5)	6.46	817.68	7.11	4.84	6.18	6.96	218.37	6.54
(1,7)	11.12	1018.82	10.44	8.24	11.21	11.63	301.16	12.17
10 pd/P.S.#1								
(1,1) /	1.43	8.55 /	8.50	1.33 /	1.43	8.55 /	8.50	1.33 /
(1,3)	4.64	31.55	8.52	1.42	4.52	10.49	174.01	6.94
(1,5)	14.60	22.57	11.85	2.47	10.38	12.63	95.46	16.33
(1,7)	31.56	28.37	16.06	5.11	19.07	17.94	110.22	29.44
10 pd/P.S.#2								
(1,1) /	2.16	1.37 /	5.68	21.72 /	2.16	1.37 /	5.68	21.72 /
(1,3)	4.23	690.49	6.05	5.98	11.97	2.74	299.99	8.54
(1,5)	12.93	1096.97	8.46	18.61	11.42	5.32	813.02	24.46
(1,7)	25.51	3891.76	11.50	37.00	20.41	9.15	2402.21	48.14

a: A joint aggregate demand shock of one unit in the initial period is assumed for the Nash "noncooperative" equilibria is assumed.

Table 5 indicates that there is no one clear preferred strategy for both time horizons and degrees of "openness". With a five period horizon, policymakers are generally better off matching policies as "wage contracting profiles" become more disparate. For a large degree of "openness", matched policy strategies are indicated for all "wage contracting profile" couplets. For a small degree of "openness" (P.S.#1), matched policy strategies are optimal for more asymmetric "wage contracting profiles", while monetary policy is a "dominant strategy" with more symmetric aggregate contract lengths. Under the ten period horizon, matched policy strategies are preferred for P.S.#1 from $(\gamma,\gamma^*) = (1,1)$ to $(1,5)$. For $(\gamma,\gamma^*) = (1,7)$, fiscal policy is a "dominant strategy" for both the domestic and foreign countries. With a larger degree of "openness" (P.S.#2), monetary policy is a "dominant strategy" for the domestic country and fiscal policy is a "dominant strategy" for the foreign country as "wage contracting profiles" become more asymmetric (i.e., γ=1 and γ^*=5,7). The short contract length country obtains lower quadratic loss values by relying on fiscal policy, while the long contract length country is better off by using monetary policy. Table 6 interprets these results given in Table 5.

Table 6

	(γ,γ^*)	Domestic Country	Foreign Country
5 pd/P.S.#1	(1,1)	Monetary	Monetary
	(1,3)	M.P.(F)	Monetary
	(1,5)	M.P.(F)	M.P.(M)
	(1,7)	M.P.(F)	M.P.(M)
5 pd/P.S.#2	(1,1)	M.P.(F)	M.P.(F)
	(1,3)	M.P.(F)	M.P.(F)
	(1,5)	M.P.(F)	M.P.(M)
	(1,7)	M.P.(F)	M.P.(M)
10 pd/P.S.#1	(1,1)	M.P.(F)	M.P.(F)
	(1,3)	M.P.(F)	M.P.(M)
	(1,5)	M.P.(F)	M.P.(M)
	(1,7)	Fiscal	Fiscal
10 pd/P.S.#2	(1,1)	A.P.(F)	A.P.(F)
	(1,3)	M.P.(M)	Fiscal
	(1,5)	Monetary	Fiscal
	(1,7)	Monetary	Fiscal

M.P.≡ "Matched Policy" : a country uses the same policy instrument as that used by the opponent;
A.P.≡ "Alternative Policy": a country uses a different policy instrument than that used by the opponent;
(F): fiscal policy is the preferred instrument;
(M): monetary policy is the preferred instrument.

Within the class of "matched policy" outcomes, one can use Table 5 to determine the policy preferences (fiscal versus monetary policy) of the and foreign countries. These results are indicated by the parenthetical designation in Table 6. Under the short horizon problem, one finds that the domestic country has a distinct preference for fiscal policy, while the foreign country prefers monetary policy (both played as matched games) as "wage contracting profiles" become more asymmetric. For the "wage contracting" couplets — $(\gamma,\gamma^*) = (1,5)$ & $(1,7)$ — both countries prefer matched games, but there exists a disagreement over which policy should be used in the matched game. The foreign country would like it to be a monetary game and the domestic country prefers it to be a fiscal game. Shifting to a longer horizon (10 period), some policy disagreements still exist. With a high degree of "openness" (P.S.#2), one finds that the foreign country will pursue a fiscal policy regardless of the domestic policy instrument, while the domestic country will use monetary irrespective of the foreign policy instrument (for larger "wage contracting profile" asymmetries).

The results in Table 6 indicate which policy instruments are most effective in minimizing the country's price and output volatility given the other country's instrument. In general, a policy instrument will be preferred if it can generate an exchange rate appreciation. Following an initial positive aggregate demand shock, an appreciation of its currency will decrease the pressure on aggregate demand and will reduce the pressure on price increases. Table 6 indicates that the preferred policy instruments generally differ between the short and long contract length countries. This model has nothing to say about the manner in which these policy disagreements are to be resolved. Rather, it does posit wherein particular incentives exist regarding the choice of policy instruments.

E. Stackelberg "leader–follower" Advantages

The industrial organization literature has spent some time considering the question of whether firms have any incentive (in terms of yielding greater profits) to play the role of Stackelberg "leader" or "follower" relative to other firms in establishing some quantity of output level or some price for its good. The answer seems to depend upon whether firms are competing in price space or output space, and on whether the goods which the firms produce are "substitutes" or "complements". The intuition here is that a firm would prefer to be "leader" if it is able to make some preemptive move relative to the competing firm, while Stackelberg "follower" is the desired role if the firm is able to copy or undercut the "leader" profitably. The preemptive incentive is demonstrated in Dixit [1980] when an incumbent firm invests in excess capacity as a deterrence to entry, while the latter can occur when an entrant undercuts the price of the incumbent as in the contestable market literature (Baumol [1982]).[11] By comparing quadratic loss outcomes for the Stackelberg "leader" and "follower" within the context of this two country model, this section attempts to extend the intuition derived from the industrial organization literature into the international policy coordination framework. The focus here is on whether "wage contracting profile" asymmetries yield "leader" or "follower" advantages to either the short or long contract length countries.

Attention again centers upon the short horizon game (5 periods). Table 7 presents the quadratic loss results for domestic "leader"(SL)–foreign "follower"(SF*) and foreign "leader" (SL*)–domestic "follower"(SF) games.

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[11]See Gal–Or [1985]

Table 7[a]

(γ,γ^*)	Domestic Leader (SL)	Foreign Follower (SF*)	Domestic Follower (SF)	Foreign Leader (SL*)
Fiscal/P.S.#1				
(1,1)	0.5091	3.6273	3.6273	0.5091
(1,3)	0.9031	10.3685	0.7541	8.9522
(1,5)	1.9943	22.0322	1.7842	19.3969
(1,7)	4.0587	38.3685	3.6352	34.1932
(1,9)	7.0819	59.3663	6.3336	53.2691
Fiscal/P.S.#2				
(1,1)	1.4391	1.1529	1.1529	1.4391
(1,3)	2.6752	2.9417	2.5731	3.0463
(1,5)	4.8365	6.4207	4.8536	6.2190
(1,7)	8.1809	11.9618	8.1633	11.6545
(1,9)	12.6400	19.9720	12.5514	19.6152
Monet./P.S.#1				
(1,1)	1.3804	1.3312	1.3312	1.3804
(1,3)	2.5180	2.9761	2.5277	2.9902
(1,5)	4.5620	5.9726	4.5758	5.9642
(1,7)	7.5871	10.7445	7.5676	10.7789
(1,9)	11.5553	17.5003	11.5248	17.6121
Monet./P.S.#2				
(1,1)	1.8960	1.4803	1.4803	1.8960
(1,3)	3.6171	2.8346	3.2253	3.1029
(1,5)	6.8575	5.8246	6.1392	6.1281
(1,7)	12.3727	10.4898	10.7864	11.1936
(1,9)	20.2911	16.7679	17.3090	18.3354

a: A joint aggregrate demand shock of one unit in the initial period is assumed, along with a five period horizon.

Using Table 7, the domestic country would prefer to be a Stackelberg "leader" ("follower") if SL <(>) SF. Similarly, the foreign country would prefer to be a Stackelberg "leader" ("follower") if SL* <(>) SF*.

Under the Fiscal games one finds that the long contract length country (domestic country) is better off playing the "follower" role and the short contract length country (foreign country) the Stackelberg "leader" role as "wage contracting profiles" become more asymmetric ($\gamma^* \geq 7$). The foreign "leader"–domestic "follower" preference exists for all asymmetric "wage contracting profiles" in P.S.#1, but only appears in P.S.#2 for greater "profile" asymmetries. The foreign country has an incentive to precommit to some fiscal policy stance in every period (given that it understands the domestic country's optimization problem), while the domestic country is perfectly willing to allow this to occur. The domestic country can minimize its losses by reacting to the foreign country's policy decision. Hence, there exists a well–defined Stackelberg equilibrium. This is illustrated in Figures 24 and 25 for $(\gamma,\gamma^*) = (1,7)$.

Figure 24: Stackelberg Equilibrium for a five period Fiscal game (P.S.#1)

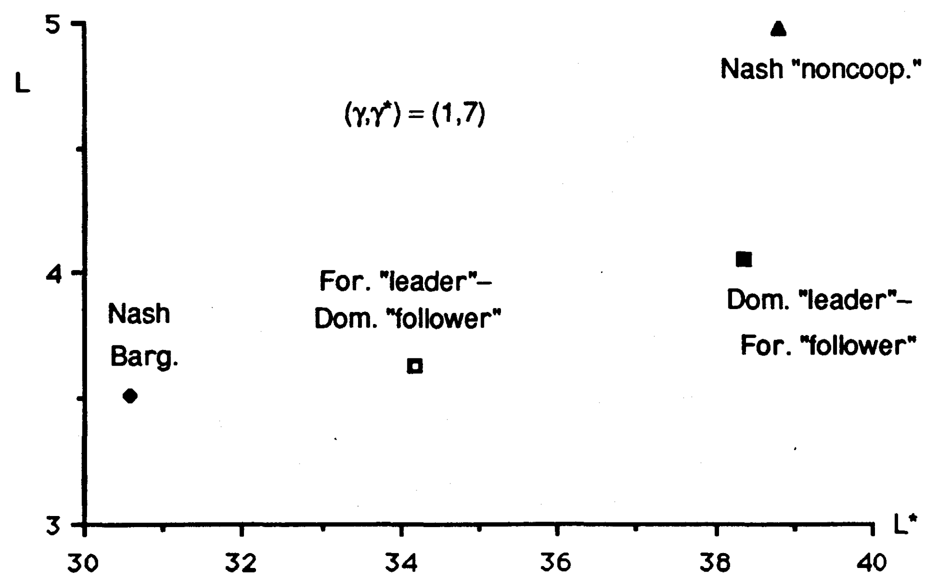
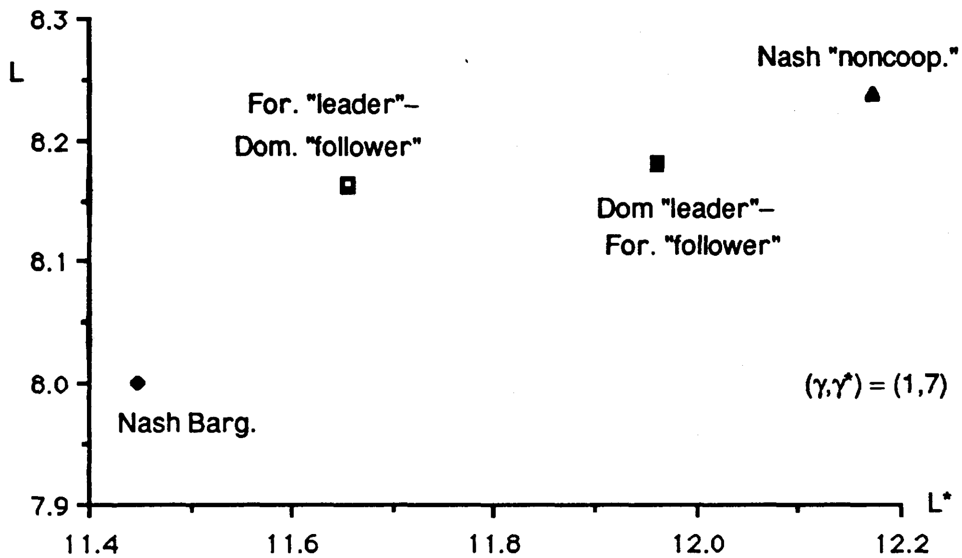


Figure 25: Stackelberg Equilibium for a five period Fiscal game (P.S.#2)



Figures 24 and 25 place the two Stackelberg equilibria with the Nash "noncooperative" and "bargaining" solutions. In each case, the domestic "follower"–foreign "leader" Stackelberg equilibrium (white square) is the preferred Stackelberg outcome, both of which dominate the Nash "noncoöperative" result and neither of which dominate the "bargaining" solution.

In the Monetary games, with larger "wage contracting profile" asymmetries, the domestic and foreign countries both have incentives to play the role of Stackelberg "follower". Neither player has an incentive to be the first mover in the Monetary game, but rather, would prefer to react to its opponent's monetary stance. This result is illustrated in Figures 26 and 27 for $(\gamma,\gamma^*) = (1,7)$.

Figure 26: Stackelberg Equilibrium for a five period Monetary game (P.S.#1)

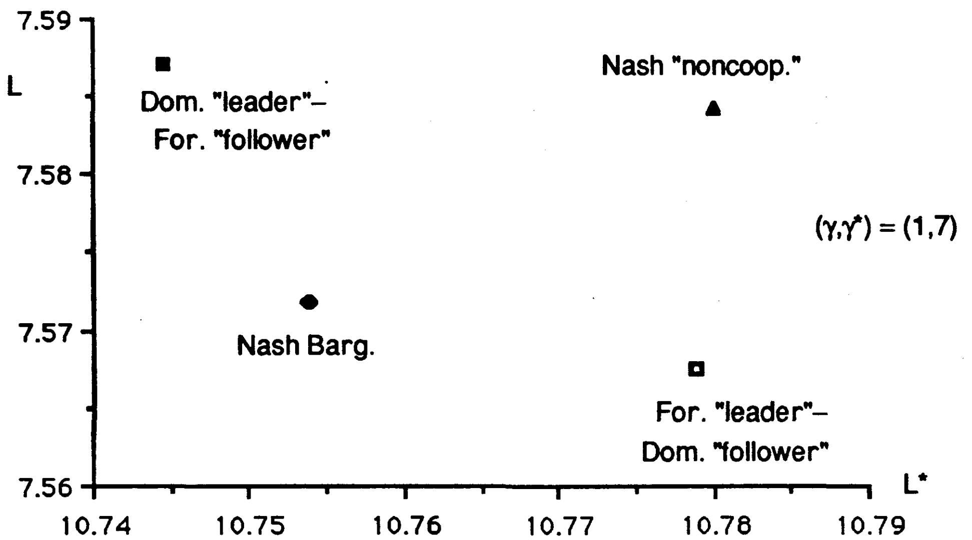
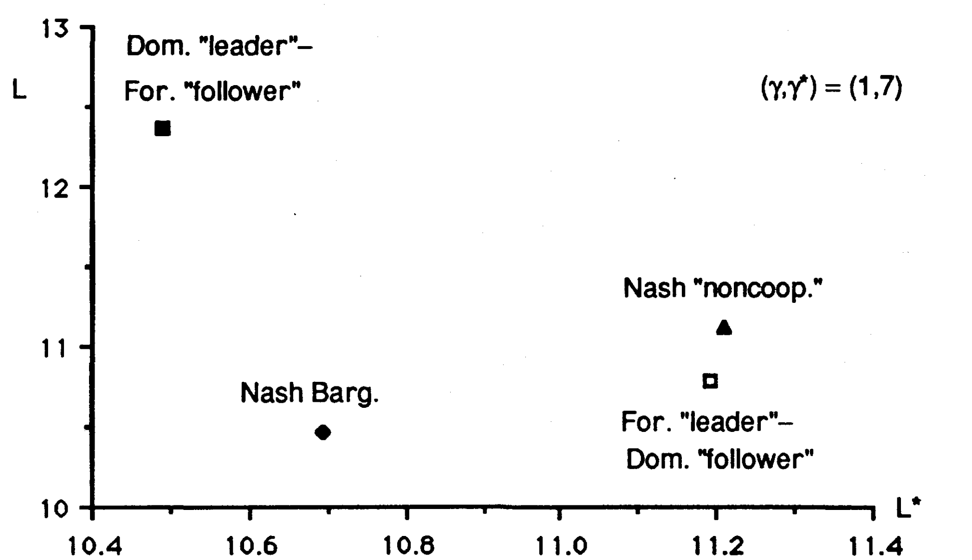


Figure 27: Stackelberg Equilibrium for a five period Monetary game (P.S.#2)



There does not exist a well–defined Stackelberg equilibrium for either Monetary game. With each player attempting to be a "follower" the most intuitive resolution of this game is the Nash "noncooperative" equilibrium. Note that if the foreign country was able to play the "follower" role (black welfare gains beyond those obtained in the "bargaining" equilibrium for both P.S.#1 and P.S.#2. The same holds for the domestic country only for P.S.#1. However, allowing one country to be the "follower" would impose large quadratic losses on the "leader", and so, not a reasonable outcome of the Monetary game.

Drawing upon the intuition from the industrial organization literature, a policymaker will prefer to be a "follower" if the available policy instrument enables the policymaker to offset any policy action by the other country. The Stackelberg "leader" has the first policy move in the game and incorporates the "follower's" policy reaction into his own optimization problem. A policymaker with an ineffective policy instrument may prefer to use this additional information regarding the opponent's policy reaction function and precommit to some policy stance. This is consistent with the outcomes for the Fiscal games under P.S. #1 and P.S. #2, where the fiscal instrument is the preferred instrument by the domestic policymaker for the 5 period horizon under the Nash "noncooperative" equilibrium. From Figures 24 and 25 we see that there is a well–defined Stackelberg equilibrium in which the domestic country is the "follower" and the foreign country is the "leader". Under the Monetary games, there is no well–defined Stackelberg equilibrium for either P.S. #1 or P.S. #2. Again, using the Nash "noncooperative" outcomes as a guide to an instrument's effectiveness, we know that the domestic country does marginally better than the foreign country (see Table 4). However, monetary policy is the preferred policy instrument of the foreign policymaker. In this case each player wishes to be a Stackelberg "follower".

SECTION 5: CONCLUSIONS

This paper has looked at a number of issues regarding the impact of differing "wage contracting profiles" on the policy strategies and macroeconomic performance of two countries engaged in a multi–period dynamic game. In general, it is found that both countries experience welfare losses as the contract lengths in one of the countries shortens. The additional volatility in prices and output are shared internationally. In addition, for games in which policymakers operate under a short horizon, the greater inertia present in the long contract country is a desirable feature as measured by the game–theoretic welfare outcomes. However, lengthening the policymakers' time horizon results in more advantageous welfare outcomes for the short contract country.

In most cases, the long contract country has a much stronger bargaining position relative to the short contract country in forming a cooperative agreement. Under the short horizon, Fiscal games result in greater welfare gains from bargaining for the short contract country at the expense of its own policy goals in the cooperative objective. Whether the short contract country is willing to pay such a price for a cooperative agreement is an open question. The bargaining position of the short contract country is improved in the Monetary games, but the gains from bargaining are lower and are diminishing as "wage contracting profiles" become more asymmetric. These results are supported by fact that, under a short horizon, the long contract country prefers to play a Fiscal game while the short contract country prefers a Monetary game. The identification of dominant strategies suggest that the institutional contracting asymmetries do form differing views on the effectiveness of policy instruments in the respective countries.

Finally, in a short horizon Fiscal game, there is a well-defined Stackelberg equilibrium where the short contract country prefers the Stackelberg "leader" role and the long contract country is better off as the "follower". This is contrasted with the Monetary game in which both players prefer the "follower" role. In general, a preference for the "follower" role infers some ability to effectively counteract moves made by the "leader" with the policy tool at hand. This may explain why the short contract country has lower welfare losses as the Stackelberg "leader" in the Fiscal game and as the "follower" in the Monetary game, since monetary policy is its preferred policy tool under the short horizon.

In addition to the results summarized above, this paper employs a more appealing wage contracting process. In contrast to much previous work which assumes that current wage demands are based on the past values of the relevant variables or on their current values alone, this paper posits rational agents who make their wage demands based on the future evolution of the variables to which they attach importance for the period in which the contract will be in effect. This enables one to capture the different horizons which wage-setters have in the long contract country as opposed to the short contract country. Given this more realistic modeling of wage-setting behavior, this paper has provided some insight into the effect that "wage contracting profile" asymmetries (differences in labor markets) have on the formation of policy in an interdependent world.

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Appendix A

1. Nash Equilibrium: Recursive Solution

In order to validate the general form recursive solution for the Nash equilibrium, the recursive technique is extended for two periods. The general recursive form can then be easily detected.

Period T–1

In period T–1, the final period in which policy is formed, the domestic policymaker minimizes the objective function for period T. The one–period dynamic programming problem takes the form:

$$\text{Min}_{\{u_{T-1}\}} \qquad V_T = x'_T K x_T + E_{T-1}(\bar{V}_{T+1})$$

$$\text{subject to} \quad x_T = A x_{T-1} + C u_{T-1} + C^* u^*_{T-1} + F v_{T-1}$$

where $\bar{V}_{T+1}$ is the optimal value of the problem solved in period T. It is assumed that $\bar{V}_{T+1} = 0$ since the current government cares not for its country's welfare beyond the T periods of the game. Substituting the constraint, the policymaker's problem becomes:

$$(1N) \quad \mathrm{Min}_{\{u_{T-1}\}} \quad [A\,x_{T-1} + C\,u_{T-1} + C^*\,u^*_{T-1} + F\,v_{T-1}]'\,D^N_{T-1}$$

$$[A\,x_{T-1} + C\,u_{T-1} + C^*\,u^*_{T-1} + F\,v_{T-1}]$$

where $D^N_{T-1} = K$.

The first–order condition is:

$$\frac{\partial V_T}{\partial u_{T-1}} = 2\,C'\,D^N_{T-1}\,[A\,x_{T-1} + C\,u_{T-1} + C^*\,u^*_{T-1} + F\,v_{T-1}] = 0$$

The domestic reaction function for period T–1 is then:

$$(2N) \qquad u_{T-1} = -\,G^N_{T-1}\,[A\,x_{T-1} + F\,v_{T-1} + C^*\,u^*_{T-1}]$$

where $\qquad G^N_{T-1} = [C'\,D^N_{T-1}\,C]^{-1}\,C'\,D^N_{T-1}$

Similarly, the foreign reaction function for period T–1 is:

$$(3N) \qquad u^*_{T-1} = -\,G^{*N}_{T-1}\,[A\,x_{T-1} + F\,v_{T-1} + C\,u_{T-1}]$$

where $\qquad G^{*N}_{T-1} = [C^{*\prime}\,D^{*N}_{T-1}\,C^*]^{-1}\,C^{*\prime}\,D^{*N}_{T-1}$

$$D^{*N}_{T-1} = K^*$$

Substituting equation (3N) into equation (2N), the Nash policy feedback rule for the domestic country at time T–1 is:

$$(4N) \quad u_{T-1} = -(R^{N}_{T-1})^{-1} S^{N}_{T-1} [A x_{T-1} + F v_{T-1}]$$

where $$R^{N}_{T-1} = I - \{ [G^{N}_{T-1} C^{*}][G^{*N}_{T-1} C]\}$$

$$S^{N}_{T-1} = G^{N}_{T-1} [I - C^{*} G^{*N}_{T-1}]$$

Substituting equation (2N) into equation (3N) yields the Nash policy feedback rule for the foreign country at time T–1:

$$(5N) \quad u^{*}_{T-1} = -(R^{*N}_{T-1})^{-1} S^{*N}_{T-1} [A x_{T-1} + F v_{T-1}]$$

where $$R^{*N}_{T-1} = I - \{ [G^{*N}_{T-1} C][G^{N}_{T-1} C^{*}]\}$$

$$S^{*N}_{T-1} = G^{*N}_{T-1} [I - C G^{N}_{T-1}]$$

Thus, the Nash equilibrium policy feedback rules depend upon the current values of the state variables and the current values of the random shocks. Substituting equations (4N) and (5N) into equation (1N) yields the optimal value of the domestic country's time T quadratic loss function:

$$\bar{V}_{T} = (A x_{T-1} + F v_{T-1})' [\Omega'^{N}_{T-1} D^{N}_{T-1} \Omega^{N}_{T-1}] (A x_{T-1} + F v_{T-1})$$

where $$\Omega^{N}_{T-1} = I - C (R^{N}_{T-1})^{-1} S^{N}_{T-1} - C^{*} (R^{*N}_{T-1})^{-1} S^{*N}_{T-1}$$

Substituting the Nash feedback rules into the foreign country analogue to equation (1N) yields:

$$\bar{V}^*_T = (A\,x_{T-1} + F\,v_{T-1})'\,[\,\Omega'^{N}_{T-1}\,D^{*N}_{T-1}\,\Omega^{N}_{T-1}\,]\,(A\,x_{T-1} + F\,v_{T-1})$$

Period T–2

Moving backward one period from period T–1, consider the domestic policymaker's optimization problem at time T–2:

$$\mathrm{Min}_{\{u_{T-2}\}} \qquad V_{T-1} = x'_{T-1}\,K\,x_{T-1} + E_{T-2}(\bar{V}_T)$$

$$\text{subject to} \quad x_{T-1} = A\,x_{T-2} + C\,u_{T-2} + C^*\,u^*_{T-2} + F\,v_{T-2}$$

$$E_{T-2}(\bar{V}_T) = x'_{T-1}\,A'\,[\,\Omega'^{N}_{T-1}\,D_{T-1}\,\Omega^{N}_{T-1}\,]\,A\,x_{T-1}$$

The optimization problem can then be rewritten as:

$$\mathrm{Min}_{\{u_{T-2}\}} \quad [\,A\,x_{T-2} + C\,u_{T-2} + C^*\,u^*_{T-2} + F\,v_{T-2}\,]'\;D^{N}_{T-2}\;[\,A\,x_{T-2} + C\,u_{T-2} + C^*\,u^*_{T-2} + F\,v_{T-2}\,]$$

where $D^{N}_{T-2} = K + [\,\Omega'^{N}_{T-1}\,D^{N}_{T-1}\,\Omega^{N}_{T-1}\,]$.

The resulting first–order condition for period T–2 is:

$$\frac{\partial V_{T-1}}{\partial u_{T-2}} = 2\,C'\,D^N_{T-2}\,[\,A\,x_{T-2} + C\,u_{T-2} + C^*\,u^*_{T-2} + F\,v_{T-2}\,] = 0$$

The domestic country's reaction function takes the form:

$$u_{T-2} = -\,G^N_{T-2}\,[\,A\,x_{T-2} + F\,v_{T-2} + C^*\,u^*_{T-2}\,]$$

where $\quad G^N_{T-2} = [\,C'\,D^N_{T-2}\,C\,]^{-1}\,C'\,D^N_{T-2}$

Likewise, the foreign country's reaction function is:

$$u^*_{T-2} = -\,G^{*N}_{T-2}\,[\,A\,x_{T-2} + F\,v_{T-2} + C\,u_{T-2}\,]$$

where $\quad G^{*N}_{T-2} = [\,C^{*\prime}\,D^{*N}_{T-2}\,C^*\,]^{-1}\,C^{*\prime}\,D^{*N}_{T-2}$

$$D^{*N}_{T-2} = K^* + A'\,[\,\Omega'^{N}_{T-1}\,D^{*N}_{T-1}\,\Omega^N_{T-1}\,]\,A$$

The resulting Nash equilibrium feedback policy rules for the domestic and foreign countries in period T–2 are:

$$u_{T-2} = -(R^N_{T-2})^{-1} S^N_{T-2} [A x_{T-2} + F v_{T-2}]$$

where
$$R^N_{T-2} = I - \{ [G^N_{T-2} C^*] [G^{*N}_{T-2} C] \}$$

$$S^N_{T-2} = G^N_{T-2} [I - C^* G^{*N}_{T-2}]$$

$$u^*_{T-2} = -(R^{*N}_{T-2})^{-1} S^{*N}_{T-2} [A x_{T-2} + F v_{T-2}]$$

where
$$R^{*N}_{T-2} = I - \{ [G^{*N}_{T-2} C] [G^N_{T-2} C^*] \}$$

$$S^{*N}_{T-2} = G^{*N}_{T-2} [I - C G^N_{T-2}]$$

The optimized values of the period T–2 domestic and foreign objective functions are then:

$$\bar{V}_{T-1} = (A x_{T-2} + F v_{T-2})' [\Omega'^N_{T-2} D^N_{T-2} \Omega^N_{T-2}] (A x_{T-2} + F v_{T-2})$$

where
$$\Omega^N_{T-2} = I - C (R^N_{T-2})^{-1} S^N_{T-2} - C^* (R^{*N}_{T-2})^{-1} S^{*N}_{T-2}$$

$$\bar{V}^*_{T-1} = (A x_{T-2} + F v_{T-2})' [\Omega'^N_{T-2} D^{*N}_{T-2} \Omega^N_{T-2}] (A x_{T-2} + F v_{T-2})$$

2. Stackelberg Equilibrium: Recursive Solution

Again solving backward from period T–1, the general recursive form can be identified after two periods.

Period T–1

The optimization problem for the foreign country in the Stackelberg game is identical to that of the foreign country in the Nash noncooperative game. The foreign country's reaction function is then:

$$(1S) \qquad u^*_{T-1} = -G^{*S}_{T-1}[A x_{T-1} + F v_{T-1} + C u_{T-1}]$$

where
$$G^{*S}_{T-1} = [C^{*\prime} D^{*S}_{T-1} C^*]^{-1} C^{*\prime} D^{*S}_{T-1}$$

$$D^{*S}_{T-1} = K^*$$

The domestic country, as Stackelberg "leader", optimizes its objective function taking into account the reaction function of the "follower". The Stackelberg "leader's" problem in period T–1 is then:

$$\text{Min}_{\{u_{T-1}\}} \quad V_T = x'_T K x_T + E_{T-1} (\bar{V}_{T+1})$$

subject to $\quad x_T = A x_{T-1} + C u_{T-1} + C^* u^*_{T-1} + F v_{T-1}$

$$E_{T-1} (\bar{V}_{T+1}) = 0$$

and equation (1S).

Substituting the constraints yields:

$$\text{(2S)} \quad \text{Min}_{\{u_{T-1}\}} \quad [A x_{T-1} + F v_{T-1} + C u_{T-1}]'$$

$$\{ [I - C^* G^{*S}_{T-1}]' D^S_{T-1} [I - C^* G^{*S}_{T-1}] \} [A x_{T-1} + F v_{T-1} + C u_{T-1}]$$

where $D^S_{T-1} = K$.

The first–order condition is:

$$\frac{\partial V_T}{\partial u_{T-1}} = 2 C' \{ [I - C^* G^{*S}_{T-1}]' D^S_{T-1} [I - C^* G^{*S}_{T-1}] \} [A x_{T-1} + F v_{T-1} + C u_{T-1}] = 0$$

Solving for u_{T-1} yields the Stackelberg "leader" policy rule for period T–1:

(3S) $$u_{T-1} = -(R^{S}_{T-1}\, C)^{-1}\, R^{S}_{T-1}\; [\, A\, x_{T-1} + F\, v_{T-1}\,]$$

where $$R^{S}_{T-1} = C'\, [\, I - C^{*}\, G^{*S}_{T-1}\,]'\, D^{S}_{T-1}\, [\, I - C^{*}\, G^{*S}_{T-1}\,]$$

Substituting the "leader's" policy rule into the "follower's" reaction function, the "follower's" policy rule is formed:

(4S) $$u^{*}_{T-1} = -\, S^{*S}_{T-1}\, [\, A\, x_{T-1} + F\, v_{T-1}\,]$$

where $$S^{*S}_{T-1} = G^{*S}_{T-1}\, \{\, I - C\, (\, R^{S}_{T-1}\, C)^{-1}\, R^{S}_{T-1}\, \}$$

Substituting equation (3S) into equation (2S), the optimal value of the period T–1 domestic objective function is:

$$\bar{V}_{T} = (\, A\, x_{T-1} + F\, v_{T-1}\,)'\, [\, \Omega'^{S}_{T-1}\, D^{S}_{T-1}\, \Omega^{S}_{T-1}\,]\, (\, A\, x_{T-1} + F\, v_{T-1}\,)$$

where $$\Omega^{S}_{T-1} = [\, I - C^{*}\, G^{*S}_{T-1}\,]\, [\, I - C\, (R^{S}_{T-1}\, C)^{-1}\, R^{S}_{T-1}\,]$$

Substituting equations (3S) and (4S) into the foreign country analogue to equation (1S) yields the optimal value of the period T–1 foreign objective function:

$$\bar{V}^*_T = (A\,x_{T-1} + F\,v_{T-1})'\;[\Omega^{*'}{}^S_{T-1}\,D^{*S}_{T-1}\,\Omega^{*S}_{T-1}]\;(A\,x_{T-1} + F\,v_{T-1})$$

where $\quad \Omega^{*S}_{T-1} = [\,I - C\,(R^S_{T-1}\,C)^{-1}\,R^S_{T-1} - C^*\,S^{*S}_{T-1}\,]$

Period T–2

Moving back one period, we begin again by deriving the foreign country's reaction function. As in the previous period, the period T–2 Stackelberg follower's optimization problem is identical to its period T–2 noncooperative Nash problem. The foreign country's reaction function is then:

(5S) $\quad u^*_{T-2} = -\,G^{*S}_{T-2}\,[\,A\,x_{T-2} + F\,v_{T-2} + C\,u_{T-2}\,]$

where $\quad G^{*S}_{T-2} = [\,C^{*'}\,D^{*S}_{T-2}\,C^*\,]^{-1}\,C^{*'}\,D^{*S}_{T-2}$

$$D^{*S}_{T-2} = K^* + A'\,[\,\Omega^{*'}{}^S_{T-1}\,D^{*S}_{T-1}\,\Omega^*(S,T-1)\,]\,A$$

Now consider the domestic country's ("leader's") period T–2 optimization problem:

$$\mathrm{Min}_{\{u_{T-2}\}} \quad V_{T-1} = x'_{T-1} K x_{T-1} + E_{T-2}(\bar{V}_T)$$

subject to $\quad x_{T-1} = A x_{T-2} + C u_{T-2} + C^* u^*_{T-2} + F v_{T-2}$

$$E_{T-2}(\bar{V}_T) = x'_{T-1} [\Omega'^S_{T-1} D^S_{T-1} \Omega^S_{T-1}] x_{T-1}$$

and equation (5S).

By substitution, the domestic minimization problem can be rewritten as:

$$\text{(6S)} \quad \mathrm{Min}_{\{u_{T-2}\}} \quad [A x_{T-2} + F v_{T-2} + C u_{T-2}]'$$
$$\{[I - C^* G^{*S}_{T-2}]' D^S_{T-2} [I - C^* G^{*S}_{T-2}]\} [A x_{T-2} + F v_{T-2} + C u_{T-2}]$$

where $\quad D^S_{T-2} = K + A' [\Omega'^S_{T-1} D^S_{T-1} \Omega^S_{T-1}] A.$

Taking the first–order condition — $\dfrac{\partial V_{T-1}}{\partial u_{T-2}} = 0$ — the Stackelberg "leader" policy rule is:

$$\text{(7S)} \quad u_{T-2} = -(R^S_{T-2} C)^{-1} R^S_{T-2} [A x_{T-2} + F v_{T-2}]$$

where $\quad R^S_{T-2} = C' [I - C^* G^{*S}_{T-2}]' D^S_{T-2} [I - C^* G^{*S}_{T-2}]$

Using equations (5S) and (7S), the Stackelberg "follower's" policy rule is:

$$u^*_{T-2} = -S^{*S}_{T-2}[A x_{T-2} + F v_{T-2}]$$

where $$S^{*S}_{T-2} = G^{*S}_{T-2} \{I - C (R^S_{T-2} C)^{-1} R^S_{T-2}\}$$

The resulting optimized objective functions for the Stackelberg "leader" and "follower" in period T–2 are:

$$\bar{V}_{T-1} = (A x_{T-2} + F v_{T-2})' [\Omega'^S_{T-2} D^S_{T-2} \Omega^S_{T-2}] (A x_{T-2} + F v_{T-2})$$

where $$\Omega^S_{T-2} = [I - C^* G^{*S}_{T-2}] [I - C (R^S_{T-2} C)^{-1} R^S_{T-2}]$$

$$\bar{V}^*_{T-1} = (A x_{T-2} + F v_{T-2})' [\Omega^{*'S}_{T-2} D^{*S}_{T-2} \Omega^{*S}_{T-2}] (A x_{T-2} + F v_{T-2})$$

where $$\Omega^{*S}_{T-2} = [I - C (R^S_{T-2} C)^{-1} R^S_{T-2} - C^* S^{*S}_{T-2}]$$

3. Cooperative Frontier: Recursive Solution

The general recursive solution for the cooperative equilibrium is derived by considering the last two periods of the cooperative game. The single optimization problem is formed by taking a weighted average (α) of the domestic and foreign quadratic loss functions.

Period T–1

The constrained minimization problem for period T–1 is:

$$\text{Min}_{\{u_{T-1}\ \&\ u^*_{T-1}\}} \quad V_T = x'_T [\alpha K] x_T + x'_T [(1-\alpha) K^*] x_T + E_{T-1}(\bar{V}_{T+1})$$

subject to $\quad x_T = A x_{T-1} + C u_{T-1} + C^* u^*_{T-1} + F v_{T-1}$

$$E_{T-1}(\bar{V}_{T+1}) = 0$$

Substituting the constraints into the objective function, the minimization problem becomes:

$$\text{Min}_{\{u_{T-1}\ \&\ u^*_{T-1}\}} \quad [A x_{T-1} + C u_{T-1} + C^* u^*_{T-1} + F v_{T-1}]' H_{T-1}$$

$$[A x_{T-1} + C u_{T-1} + C^* u^*_{T-1} + F v_{T-1}]$$

where $\quad H_{T-1} = [\alpha D^C_{T-1} + (1-\alpha) D^{*C}_{T-1}]$

$$D^C_{T-1} = K$$

$$D^{*C}_{T-1} = K^*$$

The first–order conditions with respect to u_{T-1} and $u_{T-1}{}^*$ yield:

$$u_{T-1} = -G^{C}_{T-1} [A x_{T-1} + F v_{T-1} + C^* u^*_{T-1}]$$

where $\quad G^{C}_{T-1} = [C' H_{T-1} C]^{-1} C' H_{T-1}$

$$u^*_{T-1} = -G^{*C}_{T-1} [A x_{T-1} + F v_{T-1} + C u_{T-1}]$$

where $\quad G^{*C}_{T-1} = [C^{*\prime} H_{T-1} C^*]^{-1} C^{*\prime} H_{T-1}$

Solving the first-order conditions simultaneously, the domestic and foreign policy feedback rules are:

$$u_{T-1} = -(R^{C}_{T-1})^{-1} S^{C}_{T-1} [A x_{T-1} + F v_{T-1}]$$

where $\quad R^{C}_{T-1} = I - [G^{C}_{T-1} C^*] [G^{*C}_{T-1} C]$

$$S^{C}_{T-1} = G^{C}_{T-1} [I - C^* G^{*C}_{T-1}]$$

$$u^*_{T-1} = -(R^{*C}_{T-1})^{-1} S^{*C}_{T-1} [A x_{T-1} + F v_{T-1}]$$

where $\quad R^{*C}_{T-1} = I - [G^{*C}_{T-1} C] [G^{C}_{T-1} C^*]$

$$S^{*C}_{T-1} = G^{*C}_{T-1} [I - C G^{C}_{T-1}]$$

The corresponding minimized objective function is:

$$\bar{V}_T = (A\,x_{T-1} + F\,v_{T-1})'\,[\Omega'^{C}_{T-1}\,H_{T-1}\,\Omega^{C}_{T-1}]\,(A\,x_{T-1} + F\,v_{T-1})$$

where $$\Omega^{C}_{T-1} = [\,I - C\,(R^{C}_{T-1})^{-1}\,S^{C}_{T-1} - C^*\,(R^{*C}_{T-1})^{-1}\,S^{*C}_{T-1}\,]$$

Noting that $H_{T-1} = \alpha D_{T-1} + (1-\alpha) D^*_{T-1}$, the minimized quadratic loss functions for the domestic and foreign country are:

$$\bar{V}_{d\,T} = (A\,x_{T-1} + F\,v_{T-1})'\,[\Omega'^{C}_{T-1}\,(\alpha\,D^{C}_{T-1})\,\Omega^{C}_{T-1}]\,(A\,x_{T-1} + F\,v_{T-1})$$

$$\bar{V}_{f\,T} = (A\,x_{T-1} + F\,v_{T-1})'\,[\Omega'^{C}_{T-1}\,((1-\alpha)\,D^{*C}_{T-1})\,\Omega^{C}_{T-1}]\,(A\,x_{T-1} + F\,v_{T-1})$$

Period T–2

The single optimization problem for period T–2 is:

$$\mathrm{Min}_{\{u_{T-2}\,\&\,u^*_{T-2}\}} \quad V_{T-1} = x'_{T-1}\,[\alpha\,K]\,x_{T-1} + x'_{T-1}\,[\,(1-\alpha)\,K^*\,]\,x_{T-1} + E_{T-2}\,(\bar{V}_T)$$

subject to $$x_{T-1} = A\,x_{T-2} + C\,u_{T-2} + C^*\,u^*_{T-2} + F\,v_{T-2}$$

$$E_{T-2}\,(\bar{V}_T) = x'_{T-1}\,A'\,[\Omega'^{C}_{T-1}\,(\alpha\,D^{C}_{T-1} + (1-\alpha)\,D^{*C}_{T-1})\,\Omega^{C}_{T-1}\,]\,A\,x_{T-1}$$

Substituting the constraints into the objective function and redefining terms, the optimization problem can be rewritten as:

$$\mathrm{Min}_{\{u_{T-2}\ \&\ u^*_{T-2}\}} \quad [A x_{T-2} + C u_{T-2} + C^* u^*_{T-2} + F v_{T-2}]' \, H_{T-2}$$

$$[A x_{T-2} + C u_{T-2} + C^* u^*_{T-2} + F v_{T-2}]$$

where $\quad H_{T-2} = [\alpha\, D^C_{T-2} + (1-\alpha)\, D^{*C}_{T-2}]$

$$D^C_{T-2} = K + A' [\Omega'^C_{T-1}\, D^C_{T-1}\, \Omega^C_{T-1}]\, A$$

$$D^{*C}_{T-2} = K^* + A' [\Omega'^C_{T-1}\, D^{*C}_{T-1}\, \Omega^C_{T-1}]\, A$$

The first-order conditions with respect to u_{T-2} and u^*_{T-2} yield:

$$u_{T-2} = -G^C_{T-2} [A x_{T-2} + F v_{T-2} + C^* u^*_{T-2}]$$

where $\quad G^C_{T-2} = [C' H_{T-2}\, C]^{-1}\, C' H_{T-2}$

$$u^*_{T-2} = -G^{*C}_{T-2} [A x_{T-2} + F v_{T-2} + C u_{T-2}]$$

where $\quad G^{*C}_{T-2} = [C^{*\prime} H_{T-2}\, C^*]^{-1}\, C^{*\prime} H_{T-2}$

Solving the first-order conditions for u_{T-2} and u^*_{T-2}, the optimal domestic and foreign policy rules are:

$$u_{T-2} = -(R^{C}_{T-2})^{-1}\ S^{C}_{T-2}[A\,x_{T-2} + F\ v_{T-2}]$$

where
$$R^{C}_{T-2} = I - [G^{C}_{T-2}C^{*}]\ [G^{*C}_{T-2}C]$$

$$S^{C}_{T-2} = G^{C}_{T-2}\ [I - C^{*}\,G^{*C}_{T-2}]$$

$$u^{*}_{T-2} = -(R^{*C}_{T-2})^{-1}\ S^{*C}_{T-2}[A\,x_{T-2} + F\ v_{T-2}]$$

where
$$R^{*C}_{T-2} = I - [G^{*C}_{T-2}C]\ [G^{C}_{T-2}C^{*}]$$

$$S^{*C}_{T-2} = G^{*C}_{T-2}\ [I - C\,G^{C}_{T-2}]$$

The minimized period T–2 objective function is given by:

$$\bar{V}_{T-1} = (A\,x_{T-2} + F\,v_{T-2})'\ [\Omega'^{C}_{T-2}H_{T-2}\,\Omega^{C}_{T-2}]\ (A\,x_{T-2} + F\,v_{T-2})$$

where
$$\Omega^{C}_{T-2} = [I - C\,(R^{C}_{T-2})^{-1}\ S^{C}_{T-2} - C^{*}\,(R^{*C}_{T-2})^{-1}\ S^{*C}_{T-2}]$$

Again using the fact that $H_{T-2} = aD_{T-1} + (1-a)D^{*}_{T-1}$, the minimized domestic and foreign quadratic loss functions are:

$$\bar{V}_{d\,(T-1)} = (A\,x_{T-2} + F\,v_{T-2})'\ [\Omega'^{C}_{T-2}(\alpha\,D^{C}_{T-2})\,\Omega^{C}_{T-2}]\ (A\,x_{T-2} + F\,v_{T-2})$$

$$\bar{V}_{f\,(T-1)} = (A\,x_{T-2} + F\,v_{T-2})'\ [\Omega'^{C}_{T-2}((1-\alpha)\,D^{*C}_{T-2})\,\Omega^{C}_{T-2}]\ (A\,x_{T-2} + F\,v_{T-2})$$

TESTS FOR LIQUIDITY CONSTRAINTS: A CRITIQUE

Abstract

Much empirical work has been devoted to estimating the proportion of liquidity constrained consumers (P) and the fraction of income held by these liquidity constrained consumers (L). A common feature of these studies is that P and L are taken to be constant over time. This paper attempts to determine whether this assertion is empirically justified. Using panel data, we find that P and L do vary over time with trend and cyclical components. Hence, this study shows that it is incorrect to estimate P and L as fixed parameters over time, and that P and L should instead be treated endogenously.

I. INTRODUCTION

One of the most widely debated issues in the study of consumer behavior is the validity of the "Life Cycle – Permanent Income Hypothesis" (LC–PIH). In its purest form, the LC–PIH posits the existence of rational, forward–looking agents, constrained only by expected lifetime income endowments, who smooth consumption over their entire life. Capital markets are assumed to be perfect so that agents may lend and borrow at will, and so, are never constrained from their optimal consumption plans in any period. With the addition of income uncertainty and credit rationing, consumers may be prohibited from satisfying their "desired" consumption plans, and are, as a result, "liquidity constrained".

The policy implications of such liquidity constraints on consumer behavior have recently come to the attention of researchers, especially with regard to tax policy analysis [Hubbard and Judd, 1986]. In particular, within the framework of the LC–PIH, knowledge of the proportion of liquidity constrained consumers and the fraction of total disposable income controlled by these liquidity constrained consumers are important components in estimating the effectiveness of any proposed tax policy changes.

Much empirical work has been devoted to estimating the fraction of income held by liquidity constrained consumers. Hall and Mishkin [1982] and Hayashi [1985a], among others, provide evidence on micro data, while Hayashi [1982], Flavin [1985], and Sterling [1985] are only some of the recent studies performed on aggregate time series. A common feature of these studies is that the proportion of income held by liquidity constrained consumers is taken to be a constant over time [1]. This paper attempts to determine whether this assertion is theoretically and empirically justified. We sketch a model of consumer behavior in which the probability that a consumer is liquidity constrained in the sense that his net resources fall below his desired consumption path is endogenously determined by his preferences, endowments, and the constraint imposed on his desired borrowings by the financial sector. Using both the Survey of Consumer Finance (SCF) and the Michigan Panel Study of Income Dynamics (PSID) we derive estimates of the average probability that consumers are liquidity constrained from 1968 to 1982. We find that the average probability does vary significantly over time with apparent trend and cyclical components.

The fraction of income which accrues to liquidity constrained consumers, rather than the proportion of liquidity constrained consumers, has the most important policy ramifications. Thus, we provide estimates for the fraction of total income controlled by those consumers who are more likely to be liquidity constrained.

Finally, we address the issue of the validity of existing tests for liquidity constraints, the so-called "excess sensitivity tests". We conclude that these tests suffer from serious endogeneity problems due to their treatment of the fraction of income controlled by liquidity constrained consumers . They estimate as a parameter what is, instead, a variable which is endogenous to the consumer's maximization problem.

The paper is ordered as follows: Section II presents a description of the procedures used in deriving the estimating model as well as a discussion of the results on PSID micro data. Section III discusses the limitations of existing tests for liquidity constraints and the conclusions. In addition, an Appendix is included which gives a more detailed look at the model used in Section II of this study.

II. EVIDENCE FROM PANEL DATA

This section is divided broadly into three parts. First, the theoretical underpinnings of the empirical model will be discussed. In particular, we sketch the method used in determining the fraction of liquidity constrained consumers. Also, we discuss the initial application of the derived model to the SCF data set. In the second part, we outline the procedures employed in combining the results of the SCF estimation with the available data contained in the PSID. Finally, we present estimates for both the proportion of liquidity constrained consumers (P) and the fraction of income controlled by these consumers for the period 1968–1982.

We assume that consumers maximize expected utility, subject to a resource constraint and a borrowing constraint. The model used is one developed by Jappelli [1986]. Income is equal to the sum of a deterministic component and an agent–specific component, μ_{it}. Assuming functional forms for desired consumption and for the borrowing constraint, one can write the probability that consumer i is liquidity constrained in period t as

$$(1) \qquad P_{it} = Pr\,\{\mu_{it} \leq \mu_{it}^{*} | X_t\} \;=\; F(X_{it}\, \beta_t)$$

where β_t is a vector of parameters, X_t a matrix of exogenous variables available in the SCF applicable to all individuals, and F(.) a cumulative distribution function. The product of X and β is a reduced form for available resources – including the amount that the individual is allowed to borrow from intermediaries – net of desired consumption. The ceiling μ_{it}^{*} is the level of the idiosyncratic component of income at which the consumer's desired consumption and net resources are equal. If $\mu_{it} < \mu_{it}^{*}$, consumer i is said to be liquidity constrained in period t with probability P_{it}. The reduced form for P_{it} is a function of consumer's behavior, as well as of the nature of the constraint imposed on the consumer by the financial system.

In the model, each consumer is predicted to have some probability of being liquidity constrained. This approach stands in contrast to most existing literature, where consumers are assumed to be either (a) always constrained, (b) never constrained, or (c) constrained until a certain age and not afterwards, depending essentially on the timing of their income stream. At an aggregate level, one may only observe that a certain proportion of individuals are liquidity constrained. However, the standard approach requires different models of consumption to describe the behavior of the liquidity constrained consumers (see Section III below). The framework, instead, provides a setting for studying both constrained and unconstrained consumers. As we shall see, it also implies that, in the aggregate, the probabilty of being liquidity constrained is not likely to stay constant over time.

Candidates for inclusion in the reduced–form vector equation (1) would be income, net wealth, age, and individual characteristics (education, sex, race, etc.), as well as the arguments of the borrowing constraint. A relationship such as equation (1) is intuitively appealing. In a life–cycle context, and with plausible assumptions about the reduced form for desired consumption and the nature of the borrowing constraint, one would expect to find a negative relationship between P_{it} versus age, income and wealth. Having derived values for P_{it} across individuals, we denote the mean of P_{it} in each year as P_t, i.e. $E(P_{it}) = P_t$. In what follows we assume that the average sample probability of being liquidity constraint equals the number of consumers which will find themselves constrained in the period. Thus, we will refer to P as the fraction of liquidity constrained consumers.

In a cross–section μ_{it} is not observable, but one can infer whether $\mu_{it} < \mu_{it}^*$ or $\mu_{it} > \mu_{it}^*$ from other data in the SCF. As an operational definition of liquidity constrained consumers, we follow Jappelli [1986] by using a variable in the SCF that asked consumers whether they were turned down for credit by financial institutions. The SCF represents a cross–section data set taken in 1983 which pertains to 1982. The estimated sample proportion of liquidity constrained consumers (i.e., the proportion for which we assume that $\mu_{it} < \mu_{it}^*$) from the SCF was 21 percent. This number is consistent with the 20–25 percent range of consumers who did not seem to behave in accordance to the standard LC – PIH reported by King [1985].

Having assumed that liquidity constrained consumers can be observed, the model represented by (1) can be estimated. The estimation is done in two steps. To correct for possible measurement errors, a predictive equation for income is estimated. We assume that the fitted values of the regression measure normal earnings adjusted for age and cohort effects. The second step is to conduct a logit estimation with the income estimates serving as one of the regressors [2].

The logit estimation performed on the SCF produces an estimate of β, a vector of parameters that relate the observable variables (income, age, wealth and individual characteristics) of the SCF to the threshold value μ^*. Thus, we are able to use these coefficients to compute the proportion of liquidity constrained individuals for years other than that covered by the SCF.

In order to test the proposition of the variability in the proportion of liquidity constrained consumers, we adopt the SCF parameter estimates for 1982 as the benchmark. Then we perform simulations on the PSID using the coefficients of the logit model derived from the SCF. In this way, we are able to determine estimates of P_{it}, for the years from 1968 to 1982 (waves II through

XVI of the PSID). By averaging the P_{it} values in each year, we are also able to produce estimates of P_t (t= 1968,..., 1982) for years other than 1982. These values of P_t vary from period to period, and from the benchmark year (1982), because the underlying variables vary substantially over time.

In order to carry out the above described simulation on the PSID data set, we match variables in the PSID data set with those used in the SCF. A description of the variables used in both data sets can be found in the Appendix, which also reports the values of the estimated coefficients used in the simulation. All dollar magnitudes are converted to 1982 dollars.

The results of the simulation are presented in Figure 1, where we plot the average probability of being liquidity constrained derived for 15 PSID waves. The last observation (1982) refers to wave XVI, the latest available, while the first (1968) to wave II. This measure of income is derived by using the SCF income regression coefficients on the PSID data to find a predicted value of real income. The resulting E(Pit) value is denoted by "P" in Figure 1.

In 1982 the two surveys overlap, and it is encouraging that P is 20.6 percent, while the proportion of liquidity constrained consumers found in the SCF is 21.7 percent. The difference of only one percentage point is primarily attributable to different sample characteristics and to the fact that some of the variables – specifically, the wealth variable – are not fully comparable between the two data sets [3].

The pattern of the proportion of liquidity constrained consumers is seen to follow a slightly increasing trend, and, possibly, a pro–cyclical pattern, visible especially in the first half of the sample period. It is thus apparent that P must be treated as a variable determined endogenously by demographic characteristics and economic activity rather than, as in previous studies, as a parameter to be estimated from time series regressions and/or panel data studies.

If the fraction of liquidity constrained individuals does vary over time with cyclical and trend components which depend on variables such as income, wealth and age, then the years covered by the PSID provided a good sample for this proposition. Over this period there were significant peaks and troughs in the business and income cycles. The swings in P have not been dramatic over the 15 years of the analysis. However, a t–test for the difference between the sample means in 1971 and 1982 is highly significant even at the 1 percent level. We estimate that, from the minimum of 1971, there has been an increase of approximately 3.5 percentage points in P.

A closer inspection of Figure 1 reveals that until 1973 P remains fairly stable or declines moderately, after which P increases for the remaining years of the sample period. There are two reasons that explain this pattern, and they refer to the two variables which accounts for most of the movement in P, namely, the evolution of real family income and of the age structure of the population:

i) according to the PSID survey, average real family income, after rising until 1973, remained practically constant until the end of the 1970s and then sharply declined. As a consequence, average real family income at the beginning of the 1980s was lower by some 7 percentage points than at the end of the 1960s, and by 16 percent with respect to the 1972–73 period [4]. Changes in family income reflect changes in the level of economic activity, but in part they may reflect differences in family composition [5].

ii) the age structure of the population, despite the short period of the sample, also shows a concave pattern during the 1968–1982 period, increasing until 1972 (from an average of 42.7 years in 1968 to an average of 43.1 in 1972) and decreasing afterwards to an average of 42.0 years in 1982.

In the logit model estimated on SCF data a decrease of one percentage point in the average age of the population increases the average P by about two percentage points. A reduction in real deterministic income of $4,470 in 1982 dollars (as that occurred over the 1973–1982 period) increases P by another two percentage points. It thus can be seen that the reduction of income and average household age more than accounts for the increase in P over the sample period.

In addition to the strong trend component of P, we also find evidence of cyclical movements in P. As noted above, this is particularly visible in the early part of the sample period. Employment status and income capture this component most strongly since they are very sensitive to the level of economic activity.

A caveat is in order at this point. As noted, the logit coefficients estimated on the SCF reflect not only consumption behavior, but also the nature of the constraint imposed by lenders on consumers. In using the estimated coefficients from the SCF in years other than 1982, we assume that the parameters of the consumption function, as well as those of the credit constraint, have stayed constant. While it is very reasonable to assume that consumption behavior changes very slowly at best, the hypothesis of unchanging behavior on the part of financial intermediaries can be questioned on several grounds. First, in times of deregulation, it is likely that higher competition among intermediaries would lead to an easiness of the constraint that consumers face, and to a <u>decrease</u> of P. Second, financial innovations may also be reflected in the ability of the intermediaries to process loans, and in the reduction of the costs of gathering information. Thus, innovations may also have contributed to decrease P. As an indicator of changed behavior of intermediaries, it is sufficient to note that consumer debt in recent years has increased very substantially, from 0.58 percent of personal disposable income in 1975 to more than 70 percent at the end of 1984 [Paquette, 1986].

Since the model implicitly assumes that the financial market structure in 1982 (the year to which the SCF data set pertains) holds throughout the sample period, it is likely that this simulation underestimates the proportion of liquidity constrained consumers before 1982. Unfortunately, given the nature of the PSID survey and the operational definition of liquidity constraints that we have adopted in the SCF survey, we have found no satisfactory way of incorporating "supply side" effects in the analysis. For this reason, the estimate of P that we provide should be taken as merely indicative of its true value. More generally, this exercise is only meant to provide an example of how the changing behavior of P over time could be studied, and can not account for all sources of variation that might have potentially affected P in recent years.

With these qualifications in mind, and having obtained an estimate of P, it is a simple task to compute the value of L, the fraction of income that accrues to consumers who are likely to be liquidity constrained. For this purpose we compute the average income of the consumers with $P_{it} > E(P_{it})$ for $t=1968,...,1982$, i.e. the average income of those consumers that are on average more likely to be liquidity constrained. We denote this average as Y_{Ct}. The variable L_t is then computed as $(Y_{Ct}/Y_{Nt})P_t$, where Y_{Nt} is the average income of the sample population in year t. The results of the computation are also reported in Figure 1 (line "L"). The values of L_t that we generate increase from a minimum of 12.6 percent in 1968 to a maximum of 14.9 percent in 1982. These values are similar to the estimates of the fraction of income that accrues to liquidity constrained consumers provided on panel data by Hall and Mishkin [1982] – 20 percent for the United States – and Hayashi [1985a] – 16 percent for Japan.

As expected, L is lower than P, as constrained consumers tend to control a lesser fraction of total income than do unconstrained consumers. It is also apparent that L tracks P very closely. The swing of L is less pronounced than that of P (about two percentage points from the minimum of 1968 to the maximum of 1982), but the ratio of L to P is roughly constant at 68 percent indicating that the income distribution between constrained and unconstrained consumers has not changed dramatically over the sample period. It is apparent, however, from Figure 1 that the movements in L are more volatile as compared to those of P. This is not surprising since one would expect L to be more sensitive to changes in the level of economic activity than would P. As a final note on the plausibility of the results, one can compare them with the above discussed existing empirical evidence, as well as with the theoretical simulation provided by Hubbard and Judd [1986] who, in a nonstochastic environment predict P = 23.5 percent, and L = 14.9 percent [6]. This study provides substantial empirical evidence broadly consistent with their estimates. Moreover, we show that both P and L have exhibited an increasing trend during the 1970s and early 1980s as well as a slightly cyclical component. More generally, this exercise shows that it may be incorrect to estimate L as a fixed parameter from time series regressions, since L is an endogenous variable, i.e., a variable which moves over time as a function of other variables such as income and age. In the next Section we examine the implications of this study for the existing tests of liquidity constraints, and state the conclusions.

III. IMPLICATIONS FOR EXISTING TESTS OF LIQUIDITY CONSTRAINTS

This section discusses the most common tests for liquidity constrained. These tests try to determine whether consumption is "too" sensitive to anticipated current income than implied by the LC–PIH cum rational expectations model. One test has been presented by Hall and Mishkin [1982] in the context of a panel study. Hall and Mishkin assume the existence of two groups of consumers, those following the rational expectations–lifecycle model, and the Keynesians consumers. The first group of consumers is assumed to solve an intertemporal maximization problem, in the absence of any constraint besides one that expenditure over the lifetime cannot exceed lifetime resources. By further assuming rational expectations, Hall [1978] shows that, with quadratic utility function and constant interest rate, the Euler equation for the unconstrained consumer is

$$(2) \qquad C_U = \beta_0 + \beta_1 C_{U-1} + v$$

where C_U represents consumption by the unconstrained consumers, and v is an error term incorrelated with any information available to the consumer in the current period. The second group of consumers is assumed to be liquidity constrained, and to consume its entire current disposable income

$$(3) \qquad C_C = Y_D$$

where C_C is the amount consumed by the constrained consumer. Aggregate consumption is given by

(4) $C = C_U + C_C = C_U + LY_D + v$

where L is the fraction of total disposable income held by constrained consumers. By combining (2) and (4), C can be rewritten as

(5) $C = \beta_0 + \beta_1 C_{-1} + L(Y_D - \beta_1 Y_{D-1}) + v$

Equation (5) can be estimated by a non–linear two–stage instrumental variables procedure. A more common approach in the literature is to assume a time–series specification for the process generating Y_D to substitute in (5), and perform system estimation with cross–equation restrictions. Since (5) involves consumption at two points in time, an estimate of the fraction of liquidity constrained consumers L can only be obtained by using aggregate time–series data, or individual panel data. Indeed, this is the basic approach followed by Hayashi [1982], Summers and DeLong [1984], Flavin [1985], and Sterling [1985] on United States time–series macro data, and by Hall and Mishkin [1982], and Altonji and Siow [1986] on the PSID annual survey. A recent paper by Hayashi [1985b] provides an excellent survey of current tests for liquidity constrained consumers.

All of these studies assume a constant value of L over time, but, in light of the discussion in Section II, the estimated equations are likey to suffer from serious endogeneity problems. We have in fact shown that L is not a parameter to be estimated but, rather, an endogenous variable. It results from the combination of P, and of the distribution of income among constrained and unconstrained consumers. The variable P itself is the outcome of the maximizing behavior of consumers, of the nature of the constraint imposed on consumers by the banking sector, and of the individual shocks to income in each period. In essence, L can be written as

(6) $L = L(Z)$

where Z represents the set of aggregate variables which explain the evolution of L, including wealth, income, age, the parameters of the income distribution. We cannot, however, simply substitute (6) into (5) and estimate a variable parameter model since L(Z) is derived from a model which is fundamentally different than the model which produces (5). In contrast to previous studies, we do not posit "a priori" the existence of two groups of consumers in the economy – constrained and unconstrained consumers. Rather, we model only one type of consumer who has some non–zero probability of being liquidity constrained. We may identify the differences between this model and that which constitutes the bulk of previous studies as:

i) existing tests assume two groups of consumers in the population. This assumption seems to be overly ad hoc, especially for the group of Keynesian consumers. We assume only one;

ii) in this model consumers ("ex ante") are constrained only in a stochastic sense. Only "ex post" there will be a fraction of the population that is liquidity constrained;

iii) the probability of being liquidity constrained is an endogenous variable in this study, and so, both the proportion of the population that is liquidity constrained and the fraction of total income which is controlled by constrained consumers are also endogenous.

IV. CONCLUSIONS

In general, the results for tests of liquidity constrained consumers are very mixed. However, as Hayashi [1985b] reports, estimates of L using panel data are usually more stable, precise, and uniform than are time–series estimates. The point estimates of L are broadly consistent with those of Hall and Mishkin [1982]. This may be reconciled by noting that the sample periods are relatively short, and that L problably does not vary widely in the short term. Although the estimates do vary significantly over time, they are indeed fairly stable.

In sum, we have shown using panel data that the proper treatment of L (and P) involves the modelling of them as endogenous variables, rather than as constant parameters, over time. This result has important ramifications for policy–makers. In this context, deficit financing will have multiplier effects, the magnitude of which depends importantly on the value of L and on its behavior over time. For example, the procyclical behavior of L implies that fiscal policy is more effective at those times when it is more needed, i.e. during recessions. This study has identified what we believe to be a serious oversight of the literature on tests for liquidity constrained, and one which should be considered in future work.

APPENDIX

The purpose of the Appendix is to describe the method that we use in computing $P_{it,}$ the probability that consumer i is liquidity constrained in period t. We start by estimating a logit model using as dependent variable a qualitative variable contained in the 1983 SCF. The variable reports whether consumers were turned down by financial institutions and whether they felt that, if they applied for credit, they would have been turned down. The variables in the SCF refer to 1982, and are therefore comparable with wave XVI of the Panel Study of Income Dynamics. The estimated logit model [Jappelli, 1986] is

$$\ln [Pi / (1 - \ln Pi)] = -.0099 \text{ Y AGE} -.02 \text{ WEALTH} + .136 \text{ AGE} - .00099 \text{ AGE}^2 - .56 \text{ HOWN} -.20 \text{ RACE} - .18 \text{ NE} -.43 \text{ NC} - .21 \text{ SO} - .35 \text{ AD} - .02$$

where income Y is computed from the regression

$$Y = .035 \text{ AGE} - .0004 \text{ AGE}^2 + .01 \text{ WEALTH} + .267 \text{ EMP} + .672 \text{ EWORK} + .129 \text{ SEX} + .065 \text{ EDUC} + .092 \text{ FSIZE} + 6.92$$

The variables age, homeownership (HOWN = 1 if homeowner), race (RACE = 1 if white) , sex (SEX = 1 if male) , employment status (EMP = 1 if employed), EWORK (=1 if employed and expects to work), education (EDUC = numbers of years of education), family size (FSIZE), and the regional dummies (NE, NC, SO and AD = 1 if living in the North–East, in the North–Central, in the South and in rural areas, respectively) can be found in both the SCF and in the PSID and are fully comparable in the two surveys. All variables, except wealth in the logit equation, are significant at least at the 5 percent level.

The variable Y is "family gross income from all sources earned in 1982" in the SCF and "total family money income" in the PSID survey. The only variable which is not fully comparable between the SCF and the PSID is the aggregate of family net wealth. The SCF gives in fact a detailed description of individual wealth, inclusive of financial assets, property and debt. The PSID, instead, only reports the value of the house and gives no indication of debt or financial wealth. This would seriously bias the estimates in that, if we use the PSID wealth series, we would not only underestimate individual wealth but also attribute a value of zero wealth to non–homeowning consumers. To overcome this bias, we proceed to correct the wealth variable by adding to each consumer one half of the average value of the house of each PSID wave. The value of 1/2 is derived from the SCF, where it appears that total net wealth is on average roughly 3/2 of the value of the house.

Conforming to what done in estimating the logit model in the SCF, we exclude observations relative to individuals who do not report their age as well as those who do not report their income. Each probability presented in Figure 1 is the average of more than 6,000 observations of each PSID wave. We use the deflator of private consumption to convert income and wealth in 1982 constant dollars.

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FOOTNOTES

1. Cross-section studies ostensibly would not need to make this assumption since time is not a component of the analysis. However, if the estimate is to have any relevance bejond the time period at which the cross-section was taken, one would need to assume that the estimate is constant over time.

2. See the Appendix for exposition of the regression equations used, and for further discussion of the model.

3. See the Appendix for a description of the difference between the definition of wealth in the two surveys, and for the approximation that we have used to match the two variables.

4. The average real total family income, expressed in 1982 dollars, was $24,265 in 1968–69, climbed to a maximum of $27,185 in 1972–73 and reached a minimum of $22,715 in 1981–82. Similar results are obtained when one compares the 1970 Survey of Consumer Finances with the 1977 Consumer Credit Survey and the 1983 Survey of Consumer Finances [Federal Reserve Bullettin, 1984, pp. 680–1].

5. An increase in the number of families consisting of unmarried people (including single person families) contributed to a decrease in average family size between 1973 and 1982 and may have reduced average real family income. According to the PSID, average family size was 4.36 in 1973 and declined to 2.84 in 1982.

6. These values can be found in Hubbard and Judd [1986] Table 3. They are generated by assuming an isoelastic utility function, with an elasticity of substitution in consumption of 0.10, a rate of time preference of 0.015, and the constraint that net worth must be nonnegative.

Figure 1

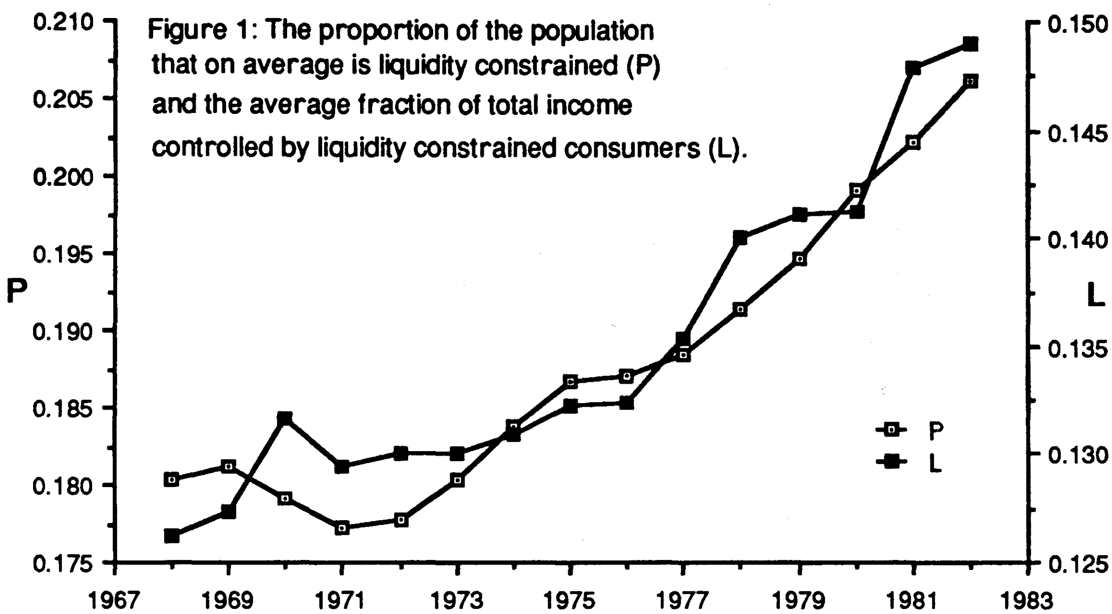
Figure 1: The proportion of the population that on average is liquidity constrained (P) and the average fraction of total income controlled by liquidity constrained consumers (L).
P
L
0.210
0.205
0.200
0.195
0.190
0.185
0.180
0.175
0.150
0.145
0.140
0.135
0.130
0.125
1967
1969
1971
1973
1975
1977
1979
1981
1983
P
L

LIQUIDITY CONSTRAINT VOLATILITY:

Evidence from Post–War U.S. Aggregate Data

Abstract

The "Life Cycle – Permanent Income Hypothesis" posits the existence of a consumer who chooses a consumption level in each period so as to maximize expected lifetime utility. Credit markets are assumed to be perfect so that the consumer may lend and borrow freely. In the absence of such credit markets, consumers may find themselves unable to satisfy their desired consumption plans, and so, are said to be "liquidity constrained". Using post–war U.S. aggregate time–series data, the extent and volatility of such credit market constraints are examined.

1. INTRODUCTION

The "Life Cycle – Permanent Income Hypothesis" (LC–PIH) maintains that rational, forward–looking individuals smooth consumption over their lifetime, constrained only by expected lifetime income endowments. It is typically assumed that capital markets are perfect so that these individuals may lend and borrow at will in order to achieve their desired consumption pattern, and so, are never constrained in any period. Recent studies, using both micro data and aggregate time–series data, have investigated the degree to which the LC–PIH characterizes the consumption behavior of individuals in an economy, and have found evidence which supports the existence of individuals who are unable to satisfy their desired consumption plans, and so, are said to be "liquidity constrained".[1]

The existence of liquidity constraints on consumers' behavior has important implications for economic policy. The effectiveness of tax policy changes and government expenditure programs would be particularly influenced by the fraction of total income held by liquidity constrained consumers in the economy. Hence, the precise measurement of these phenomena would be valuable for policymakers in formulating policy actions.

A common feature of previous studies on liquidity constrained behavior has been to treat the economy's fraction of liquidity constrained consumers as a constant over time. Using an instrumental variable technique on U.S. aggregate data from 1953:1 to 1985:4, Mankiw and Campbell [1987] estimate the fraction of liquidity constrained consumers (λ) to be between 0.35 to 0.65 depending on the

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[1] See Hall and Mishkin [1982] and Hayashi [1985] for micro data–based studies. Hayashi [1982], Flavin [1985], and Summers and DeLong [1984] present studies using aggregate time–series data.

particular instruments used. Hayashi [1982] also used an instrumental variable technique on annual U.S. data from 1948 to 1978 and obtained an estimate of λ equal to 0.892. Finally, Hall and Mishkin [1982], using U.S. panel data (the Panel Study of Income Dynamics) estimated λ as 0.20 over the 1969 – 1975 sample period. These studies, as well as those done by Flavin [1981, 1985] and Summers and DeLong [1984], conclude that the LC–PIH consumption behavior does apply to a large portion of the consumers in the U.S. economy. However, the magnitude of this deviation from the LC–PIH varies according to the sample period and the estimation technique.

Economic theory suggests that λ should be sensitive to economic and demographic variables, and thus should be treated as endogenously varying over time. Using micro data (the Survey of Consumer Finances and the Panel Study of Income Dynamics data sets), Fissel and Jappelli [1986] found that the average fraction of total income controlled by liquidity constrained consumers does vary significantly from 1968 to 1982. Fissel and Jappelli found that λ ranged from 0.126 in 1968 to 0.149 in 1982. Although not large, the movement in λ is statistically significant. This paper extends this earlier work by examining the volatility of λ over time using post–war U.S. aggregate time–series data. An examination of the endogenous movement in the fraction of liquidity constrained consumers is better handled with aggregate time–series data as compared to micro data simply due to the breadth of the available data. Moreover, the use of micro data (see Fissel and Jappelli [1986]) requires that simulations be performed to estimate the value of λ over time. In contrast, the use of aggregate time–series data allows one to use the current realizations of the necessary data in order to estimate the evolution of λ.

Consistent with the Fissel and Jappelli study using panel data, this study finds that the endogenous specification of λ significantly improves the fit of the consumption equation vis–a–vis that resulting from a parametric specification of λ. Moreover, the movement of λ over the sample period (1954:1 – 1985:4) is statistically significant. These results support the assertion that λ is properly modelled as endogenous. The remainder of the paper is outlined as follows. Section 2 presents the theoretical model, based on Hall's [1978] study. Section 3 discusses the filtering of the endogenous variables of the model, derives the estimable system of equations, and discusses the Full–Information Maximum Likelihood estimation procedure. Section 4 presents the results of the study and Section 5 offers some conclusions.

2. MODEL

Following the model developed by Hall [1978], it is assumed that, at any point in time, there exist two types of consumers in the economy — (i) liquidity constrained and (ii) unconstrained consumers . Formally, liquidity constrained consumers must devote all of their current disposable income to current consumption:

(1) $$\hat{c}_t^L = \hat{y}_t^L$$

where the " ˆ " signifies "per capita" variables, $\hat{c}_t^L$ represents consumption by a representative liquidity constrained consumer in period t, and $\hat{y}_t^L$ is the disposable income of a representative liquidity constrained consumer in period t.

The unconstrained consumer, by definition, can lend and borrow freely, and is assumed to choose his consumption level so as to maximize expected utility over his lifetime:

$$V = \max E_0 \sum_{t=0}^{T} (1+\delta)^{-t}\, U(\hat{c}_t^U)$$

$$\text{subject to } \hat{A}_{t+1} = (1+r)(\hat{A}_t + \hat{y}_t^U - \hat{c}_t^U)$$

where

$U(.) \equiv$ one–period utility function for the unconstrained consumer, assumed to be additively separable through time;
$\delta \equiv$ subjective rate of time preference;
$r \equiv$ real rate of interest, assumed constant over time $(r \geq \delta)$;
$\hat{c}_t \equiv$ consumption;
$\hat{y}_t \equiv$ income;
$\hat{A}_t \equiv$ non–human assets.

The necessary 1st–order condition for utility maximization is:

$$\text{(2)} \qquad E_t\, U'(\hat{c}_{t+1}^U) = \frac{(1+\delta)}{(1+r)}\, U'(\hat{c}_t^U)$$

Thus, for the unconstrained consumer, equation (2) relates the marginal utility of consumption in period (t+1) to the marginal utililty of consumption in period t. Assuming a quadratic one–period utility function:

$$U(\hat{c}_t^U) = \frac{1}{2}\,(\,\hat{c}_t^U - \bar{c}\,)^2$$

where $\bar{c}$ is the bliss consumption point, consumption for the unconstrained consumer in period t can be represented as:

(3) $$\hat{c}_t^U = b_0 + b_1\,\hat{c}_{t-1}^U + \hat{u}_t$$

where $b_0 = \dfrac{\bar{c}\,(r-\delta)}{(1+r)}$, $b_1 = \dfrac{(1+\delta)}{(1+r)}$, $\hat{c}_t^U$ is consumption of a representative unconstrained consumer in period t, and $\hat{u}_t$ is a "white noise" disturbance term. Equation (3) states that, for the unconstrained consumer, no information available in period t apart from $\hat{c}_t^U$ can help predict consumption in period t+1 ($\hat{c}_{t+1}^U$). Hall [1978] maintains that equation (3) is a close approximation to the stochastic behavior of unconstrained consumption under the LC–PIH.

Since aggregate time–series data are used, the aggregated analogues to equations (1) and (3) are:

(4) $$C_t^L = \lambda\,Y_t$$

(5) $$C_t^U = \beta_0 + \beta_1\,C_{t-1}^U + u_t$$

where λ is the fraction of disposable income held by liquidity constrained consumers, and Y_t is aggregate disposable income in period t. Total consumption in period t may then be written as:

$$C_t = C_t^L + C_t^U$$

(6) $$C_t = \beta_0 + \beta_1 C_{t-1}^U + \lambda Y_t + e_t$$

Since $C_{t-1}^U = C_{t-1} - \lambda Y_{t-1}$, equation (6) can be rewritten as:

$$C_t = \beta_0 + \beta_1 (C_{t-1} - \lambda Y_{t-1}) + \lambda Y_t + e_t$$

(7) $$C_t = \beta_0 + \beta_1 C_{t-1} + \lambda [Y_t - \beta_1 Y_{t-1}] + e_t$$

Assuming that disposable income may be adequately modelled as an AR(p) process:

(8) $$Y_t = \rho_0 + \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + \ldots + \rho_p Y_{t-p} + v_t$$

one has a system of two estimable equations which include the time–series process of disposable income (8) and the aggregate consumption function (7). The liquidity constraint value in equation (7) – λ – can be thought of as a shadow price on current consumption which functions as an interest rate. Whereas the unconstrained consumer can maintain normal consumption levels during periods of abnormally low income by dissaving and borrowing, the liquidity constrained consumer will reduce current consumption as if he faced a higher interest rate.[2]

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[2]See Hall [1987], p.18.

As noted above, previous work has treated λ as a constant to be estimated within this system of equations. Fissel and Jappelli [1986] show, using panel data, that λ should be modelled as endogenous, subject to other economic and demographic factors which determine its value. In this paper, I extend this insight into the context of aggregate time–series estimation. The endogeneity of λ can be represented as:

$$\text{(9)} \qquad \lambda_t = f(Z_t)$$

where Z_t is a vector of explanatory variables for the liquidity constraint variable. Two systems can now be estimated. One system will consist of equations (7) and (8) in which λ is treated as fixed (the "Constant Parameter" (CP) system, or λ_0). The other system includes equations (7), (8) and (9) where λ is determined by other macroeconomic and demographic variables (the "Endogenous Variable" (EV) system, or λ_t).

3. ESTIMATION TECHNIQUE

A. The Filtering of the Time–Series Data

This section presents the method by which the consumption and income series are filtered prior to estimation. Although frequently overlooked, the appropriate filtering of the time–series data is important. The aim is to employ a filter which transforms a nonstationary series into one which is sufficiently "whitened" so as to avoid any spurious correlations in the aggregate time–

series data.[3] This is a stronger requirement than just making the series stationary. A stochastic process is called stationary if the first two moments of the joint distributions of the finite subsequences are finite and do not change through time. Stationarity is an important property as it guarantees that there are no fundamental changes in the structure of the process that would render estimation difficult or impossible. However, a stationary process does not necessarily eliminate correlations in the data which could lead to spurious causality inferences from the estimation. These correlations are generally due to common factors, such as time, which have nothing to do with causality. It is this insight which leads Sims [1972] to filter the time series in order to flatten the spectral density.

According to Nelson and Plosser [1982], models based on time trend residuals may be misspecified if the secular movement in the time–series is of a stochastic rather than deterministic nature. This is fundamentally a question of the appropriate characterization of nonstationarity in economic time–series. A "trend–stationary" series (i.e., "detrending" a time–series by regression on a time trend) implies a deterministic time trend, while a "differenced" process implies a stochastic time trend, where it is generally assumed that the secular trend component is the source of the nonstationarity in the series (i.e., the cyclical component is stationary). Nelson and Plosser present evidence which supports the latter (stochastic trend) representation over the former (deterministic trend). Moreover, one must have some way of testing whether the filtered data is sufficiently "whitened". As noted above, a filter which renders a series stationary is not guaranteed to render it a "white noise" process.

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[3] A "white noise" process is one which is serially uncorrelated, and so, a "whitened" variable is one which is passed through a filter to make it a "white noise".

Nelson and Plosser give an example of a first difference filter which may make a series stationary, but does not remove a stochastic secular trend in the data. The results in this paper are consistent with their findings.

Before discussing the statistical results of various filters, a brief mention of the specification of the data is necessary. As noted by Mankiw and Campbell [1987], aggregate time–series data on consumption and income are more closely represented as log–linear processes than linear processes, since the mean change and the innovation variance grow with the level of the series. Hence, the scaling of these variables is necessary. The scaling strategy employed in this paper is to transform all variables to logarithmic form.

Spectral analysis is used to test for the "whitening" of the relevant time–series data. The frequency domain is able to expose aspects of the data not so readily available at the observation intervals of the time domain, especially trends and seasonality. The power spectra of the time–series data are computed and graphed. In general, the presence of a trend in the data is consistent with a large value for the spectral density function at a zero frequency (or an infinitely long period since trends do not repeat). Beyond the visual analysis of the power spectra, Durbin's Cumulated Periodogram test (see Durbin [1969]) is employed to test the "goodness–of–fit" of the filtered series against a theoretical "white noise" process, which has a constant spectral density value over the entire range. The maximum vertical distance between the two distributions yields a Kolmogorov–Smirnov statistic which can be used for testing of the "whitened" series against the null hypothesis of "white noise". The Kolmogorov–Smirnov statistics from Durbin's Cumulated Periodogram test are reported in Table 1, as applied to quarterly data over the 1947:1 – 1987:1 interval (a total of 164 observations).

Table 1

Filter	Kolmogorov–Smirnov Statistic
Income	
No Filter	0.94942
Linear Trend	0.40132
Quadratic Trend	0.37740
Cubic Trend	0.35372
4th Difference	0.67456
1st Difference	0.34088
1st –4th Difference[a]	0.06771
Consumption	
No Filter	0.94962
1st Difference	0.61153
1st –4th Difference[a]	0.09376

a: A first and fourth difference filter, i.e., the fourth difference of the first differences of the series.

The critical values for the Kolmogorov–Smirnov statistics are[4] :

90% Confidence Level: 0.1347
95% Confidence Level: 0.1502
99% Confidence Level: 0.1800

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[4] The critical values are derived from the following formulas (for a "one–sample" test):

90% Confidence Level: $1.22/\sqrt{m}$
95% Confidence Level: $1.36/\sqrt{m}$
99% Confidence Level: $1.63/\sqrt{m}$

where $m = (n/2) = 82$ (See Winkler and Hays [1975], p. 845–848).

The hypothesis test posits the "white noise" process as the null hypothesis. If the tabulated Kolmogorov–Smirnov statistic is greater than the critical Kolmogorov–Smirnov value, then the null hypothesis is rejected and the filtered process is statistically distinguishable from a "white noise" process. Looking at the income series, none of the time trend filters (linear, quadratic, or cubic) is able to reduce the income series to a "white noise" process. In addition, neither the first difference or fourth difference filters sufficiently "whiten" the income series. Only the first –fourth difference filter statistically "whitens" the income series. The latter result is consistent with the Nelson and Plosser [1982] result that first differencing does not eliminate a stochastic trend from the series. The consumption series follows the same pattern as the income series. Again, the first difference filter does not sufficiently "whiten" the series, while the first – fourth difference filter does so. The first –fourth difference filter is thus used in this paper for both the income and consumption series. The power spectra for the first and first–fourth difference filters of the income and consumption series considered in Table 1 are presented in Figures 1 – 4.[5] The power spectra for the remaining filters of the income series are presented in Appendix A.

---

[5] The "period" is given by: period = [168/entry number],while the frequency is given by : frequency = ["entry number"/168]. In addition, the spectra are computed using a window length (Parzen) of 11 (w11). The window makes the estimates smoother, and so, reduces the variance of the spectral estimate (greater efficiency) while introducing some bias in the spectral density estimate.

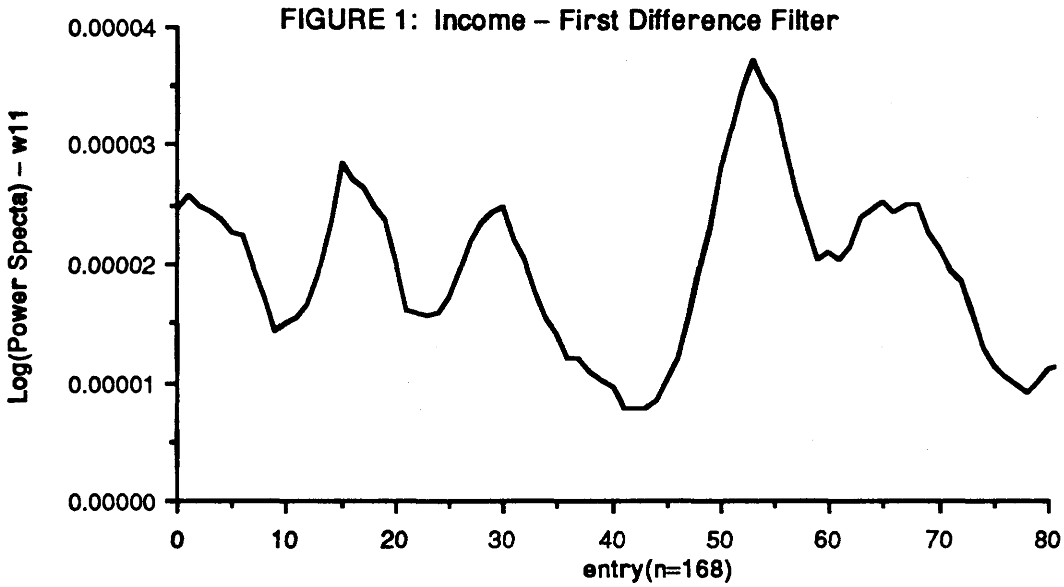


FIGURE 1: Income – First Difference Filter
Log(Power Specta) – w11
0.00004
0.00003
0.00002
0.00001
0.00000
0
10
20
30
40
50
60
70
80
entry(n=168)

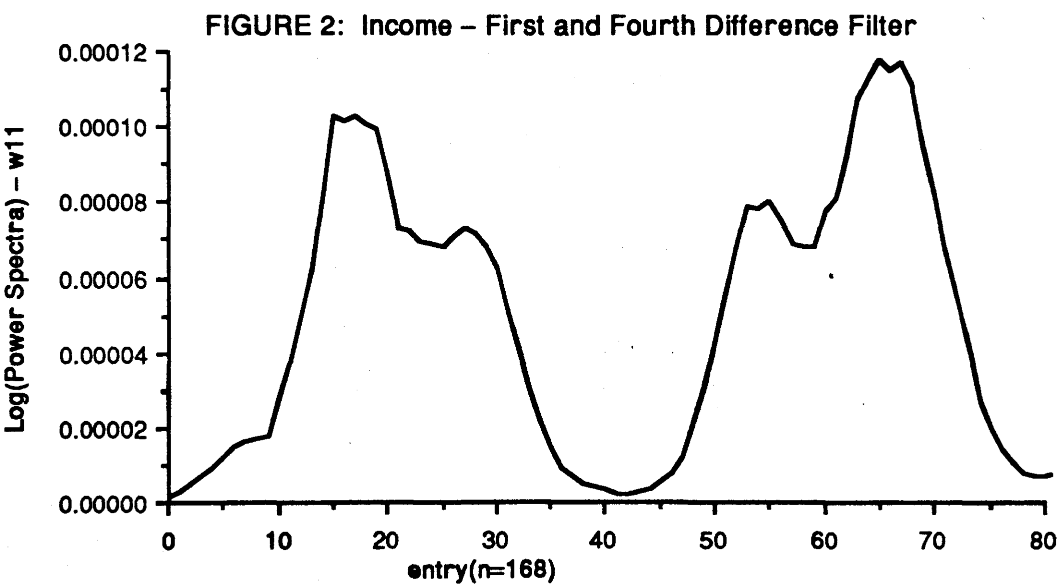


FIGURE 2: Income – First and Fourth Difference Filter
Log(Power Spectra) – w11
0.00012
0.00010
0.00008
0.00006
0.00004
0.00002
0.00000
0
10
20
30
40
50
60
70
80
entry(n=168)

FIGURE 3: Consumption – First Difference Filter

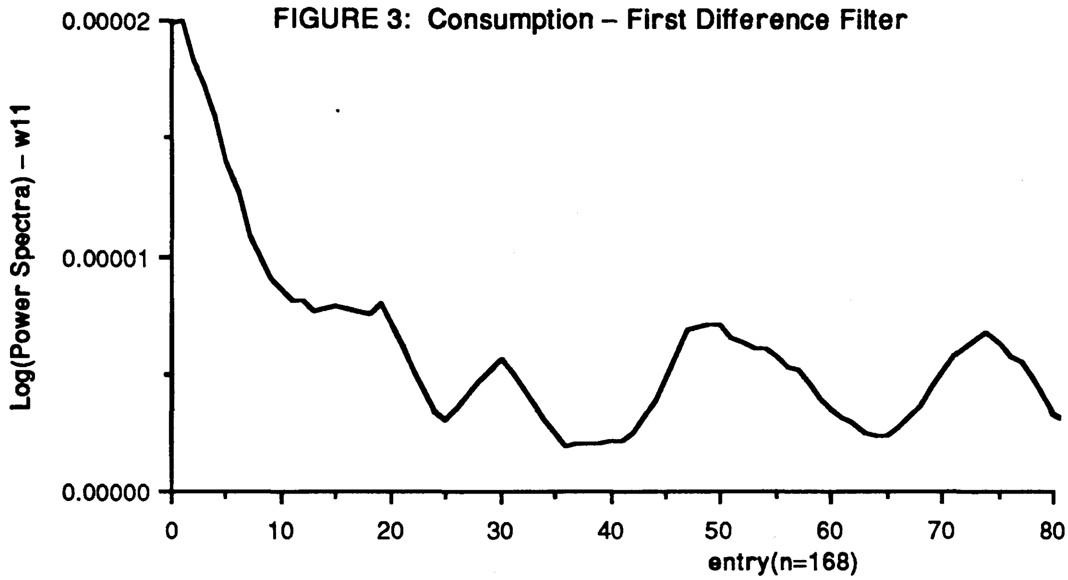
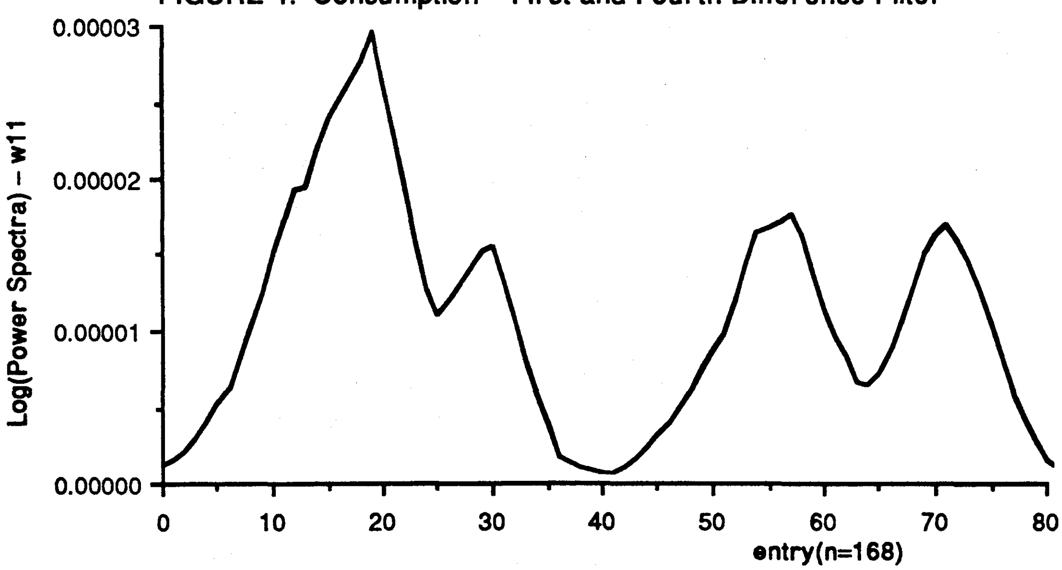


FIGURE 4: Consumption – First and Fourth Difference Filter



The evidence of a trend, as indicated by a large spectral density at the zero frequency (zero entry number), is removed by the first – fourth difference filter, but remains for the first difference filter and for the time trend filters. The first – fourth difference filtered income and consumption series (in logs) are shown in Figures 5 and 6, respectively. Figures 5 and 6 are the time domain analogues of Figures 2 and 4 which present the series in the frequency domain.

FIGURE 5: Filtered Income Series

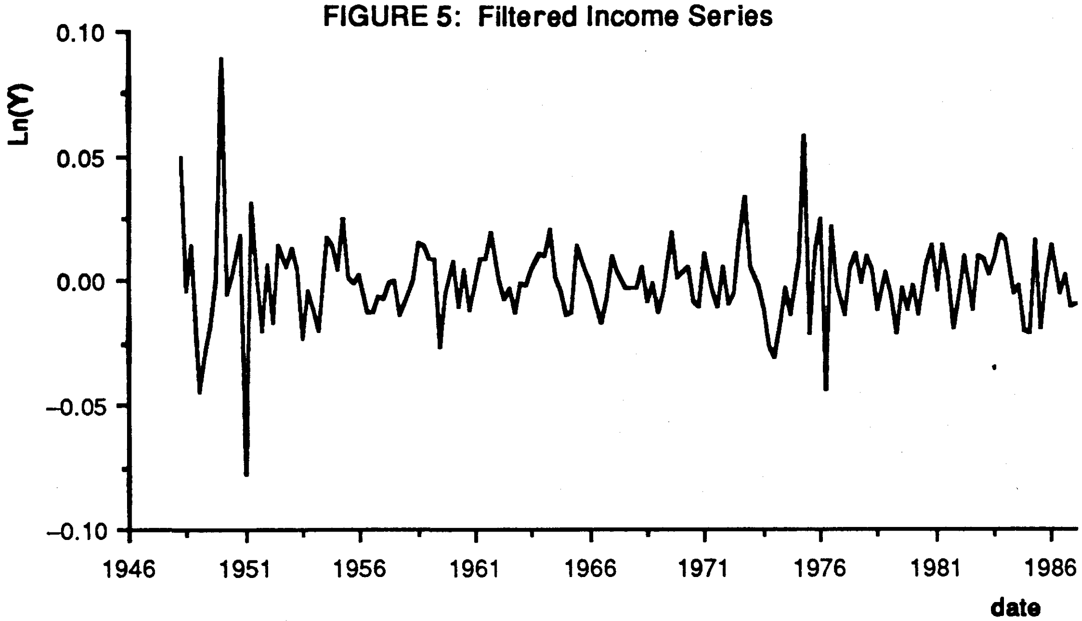
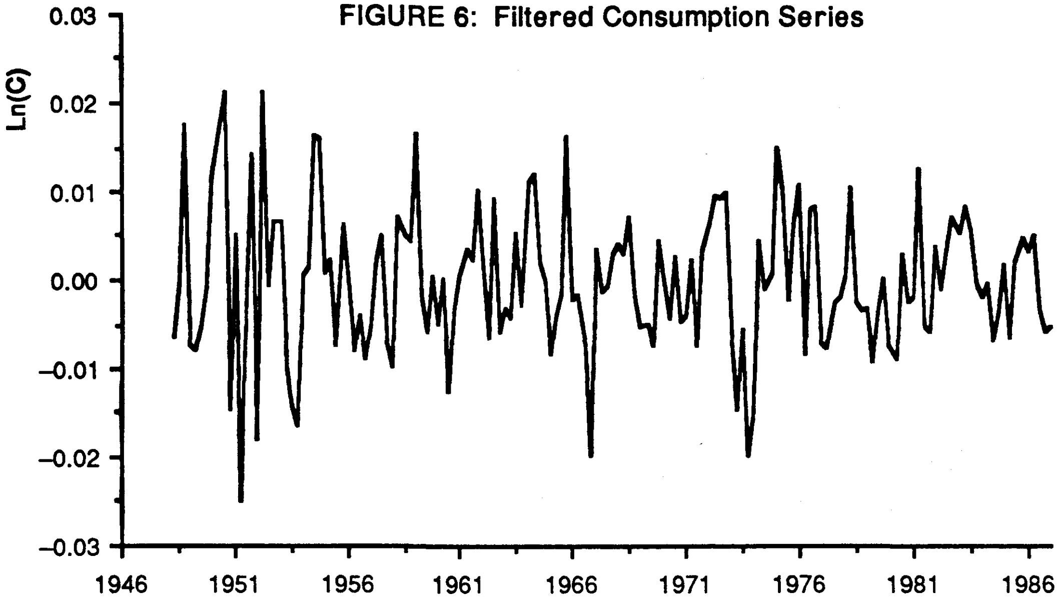


FIGURE 6: Filtered Consumption Series



B. Full–Information Maximum Likelihood Estimation

A Full–Information Maximum Likelihood (FIML) estimation technique is chosen to estimate the system of two simultaneous equations. Consider the following 2–equation nonlinear system:

(10) $$y_t = f(X_t, \beta) + u_t$$

where

$\mathbf{y}_t = (y_t^1 \; y_t^2)' \equiv$ vector of endogenous variables;

$\mathbf{X}_t = (X_t^1 \; X_t^2)' \equiv$ matrix of regressors;

$\boldsymbol{\beta} = (\beta^1 \; \beta^2)' \equiv$ vector of unknown parameters;

$\mathbf{u}_t = (u_t^1 \; u_t^2)' \equiv$ vector of error terms.

The model (10) is estimated under the normality assumption of u_t, i.e.,

$$u_t \sim N(0, \Sigma)$$

Amemiya [1977] has shown that the FIML estimator is consistent if the true distribution of u_t is normal, but is, in general, inconsistent if u_t is not normal. However, the FIML estimator is asymptotically more efficient than the non–linear three–stage estimator if u_t is normally distributed.

Amemiya [1983] and Engle [1984] have shown that the likelihood ratio test (LRT) is a proper test statistic for this hypothesis–testing problem:

$$\text{(11)} \qquad \text{LRT} = 2 [\log L(\beta) - \log L(\tilde{\beta})] \sim \chi^2(q)$$

where $L(\beta)$ is the log–likelihood for the unconstrained system, $L(\tilde{\beta})$ is the log–likelihood for the constrained system and q is the number of parameter restrictions. Under the null hypothesis that the constraints are appropriate, the LRT statistic has a chi–squared distribution with q degrees of freedom. With the FIML technique, one can use a likelihood–ratio test statistic to determine whether the advantage gained by modelling λ as an endogenous variable is statistically significant. The estimated parameters in the EV system can be used to track the movement in the fraction of income controlled by liquidity constrained consumers (λ_t) over the sample period (1954:1 – 1985:4).

Filtered income can be adequately modelled as an AR(4) process. This is statistically verified since spectral analysis of the residuals of this income generating process reveals that the residuals are "white noise". The Kolmogorov–Smirnov statistic derived from Durbin's Cumulative Periodogram test for the residuals of this AR(4) process is 0.065. As noted earlier, this value falls below the 90% confidence level, and so, the residuals are not significantly different from a "white noise" process. The estimated system is then:

(12) $$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} + \alpha_4 Y_{t-4} + v_t$$

(13) $$C_t = \beta_0 + \beta_1 C_{t-1} + \lambda (\alpha_0 + (\alpha_1 - \beta_1)Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} + \alpha_4 Y_{t-4}) + e_t$$

The EV system, where the liquidity constraint parameter – λ_t – is modelled as a value which evolves over time and is determined by other aggregate time–series variables, is tested against the standard assumption (the CP system) which treats the liquidity constraint parameter as a constant – λ_0. Formally, the alternative specifications for λ are:

(14) **CP:** $$\lambda = \lambda_0 = g_0$$

(15) **EV** $$\lambda = \lambda_t = g_0 + g_1 \bullet ur_t + g_2 \bullet age_t + g_3 \bullet dr_t + g_4 \bullet ic_t$$

where $ur_t \equiv$ log of unemployment rate;

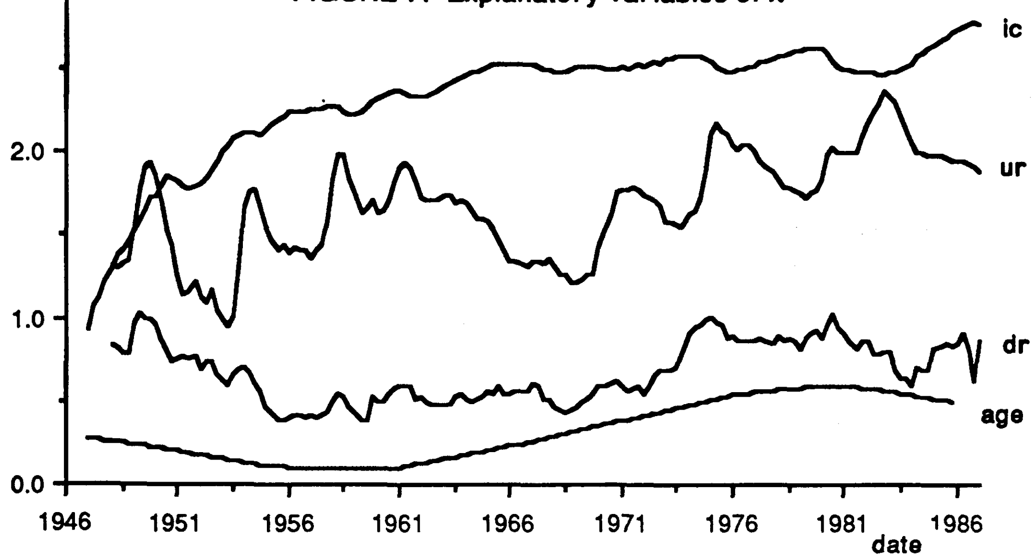
$age_t \equiv$ log of the ratio of the number of people in the 20–35 age category to the number of people in the 40–55 age category;

$dr_t \equiv$ log of the delinquency rate on consumer installment loans, 30 days or more;

$ic_t \equiv$ log of the ratio of consumer installment loans to personal income.

The variables used to determine the evolution of λ_t are aggregate economic, demographic and financial variables. The unemployment rate, which was used by Flavin [1985], proxies for the cyclical movement in economic activity in the U.S. economy over the sample period. The ratio of the number of consumers in the 20 – 35 age category to the number of consumers in the 40 – 55 age category is used to identify the relative age distribution of the U.S. population. Consistent with the Life Cycle hypothesis, this variable measures the relative number of consumers in the low–earning period of their lives to the number of consumers in the high–earning period of their lives. Two financial variables are also included — the delinquency rate of thirty days or more on consumer installment loans and the ratio of consumer installment credit to personal income. They are meant to capture the degree of credit market "tightness", as well as the general trend of credit market expansion over the sample period. The explanatory variables of λ_t are presented in Figure 7.

FIGURE 7: Explanatory variables of λ



4. RESULTS

The two equation system (equations (12) and (13)) was estimated simultaneously using a Full Information Maximum Likelihood technique while imposing the various specifications for λ: equation (14) for the CP system and equation (15) for the EV system. The sample period was from 1954:1 through 1985:4. The beginning of the sample period was chosen to avoid the Korean War which, as shown by Mankiw and Campbell [1987], has serious effects on the estimated liquidity constraint values. Table 2 contains the parameter estimates for the CP and EV systems.[6]

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[6] As noted below, 1980:1 – :2 was a period in which the government placed controls on the credit markets. The removal of these two quarters from the sample failed to improve the fit of the model.

Table 2[a]

Coefficient	CP	EV
g_0	0.356	1.65
	(4.91)	(1.47)
g_1		–0.27
		(–0.53)
g_2		–0.22
		(–0.29)
g_3		1.03
		(1.22)
g_4		–0.64
		(–1.34)
Log–Likelihood	1183.0	1226.4
R^2: Consumption Eqn.	0.67	0.76

a: "t–statistics" are reported parenthetically.

The coefficients on the unemployment rate (g_1) and on the age distribution (g_2) are not individually significant and have signs which are the opposite of that which is expected. One would expect the coefficients on the unemployment rate and the ratio of the number of consumers in their low–earning years to the number of consumers in their high–earning years to be positive, i.e., increases in these variables should raise λ_t. The former result is in contrast to the finding of Flavin [1985]. Flavin uses the unemployment rate as a proxy for the severity and prevalence of liquidity constraints on consumption and finds that the excess sensitivity of consumption to the unemployment rate is negative.[7] Of greater interest are the coefficient estimates of the delinquency rate (g_3) and the ratio of installment credit to personal income (g_4) which reflect the credit market conditions over the sample period. In addition, the delinquency rate reflects the level of economic activity in the economy. The coefficient on the delinquency rate is positive, as expected, and is marginally significant at the 80% confidence level. A greater delinquency rate on installment credit implies riskier behavior by lending institutions as well as decreased economic activity. The coefficient on the ratio of installment credit to personal income, which is a measure of credit market tightness, has the expected negative sign and is significant at the 80% confidence level. A greater ratio of installment credit implies greater credit availability in the economy.

The joint significance of the variables which explain the evolution of λ_t can be determined by using the Likelihood Ratio Test as presented in Table 3.

[7] The perverse signs on the unemployment rate and the age distribution coefficients may be due to the collinearity of these variables. Estimating the EV system without "dr" (the delinquency rate) renders a positive (but insignificant) sign for the coefficient on the age distribution; however, the sign for the coefficient on the unemployment rate remains negative. In addition, using one and two period lags on the unemployment rate yield negative and insignificant signs on their coefficients.

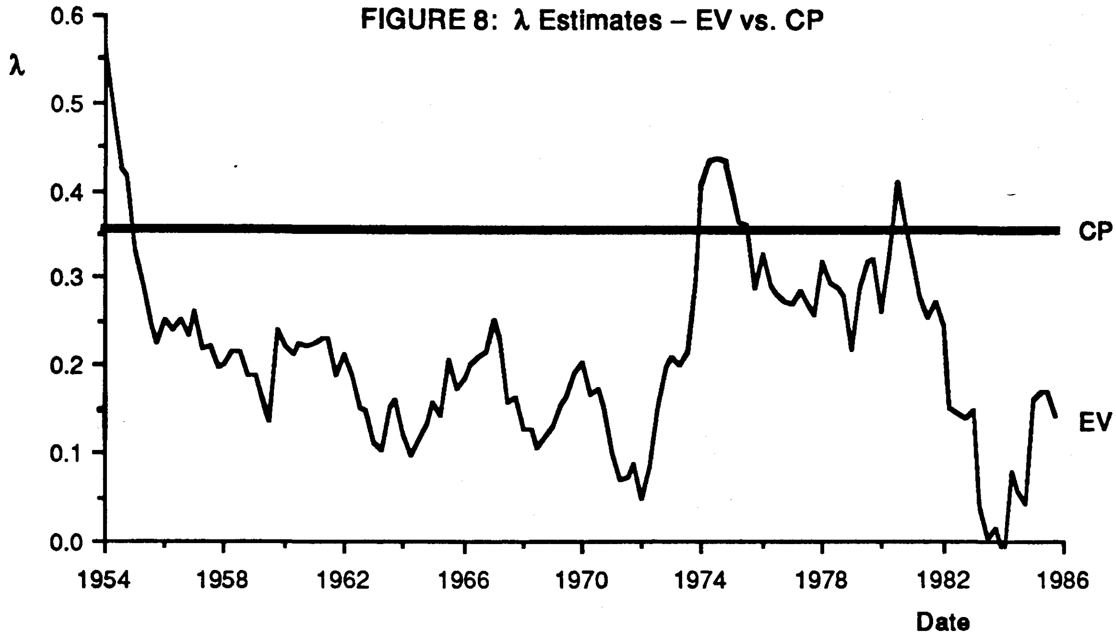
Table 3: Hypothesis Test of the EV system versus the CP system

H_0: $g_1 = g_2 = g_3 = g_4 = 0$
H_a: $g_1 \neq 0, g_2 \neq 0, g_3 \neq 0, g_4 \neq 0.$

LRT statistic = 86.8 $\Rightarrow$ reject H_0.

a: The chi–squared critical value (with a probability of 0.005 that a value beyond that critical value would be realized) for the EV system versus the CP system is: χ^2 (4) = 14.86.

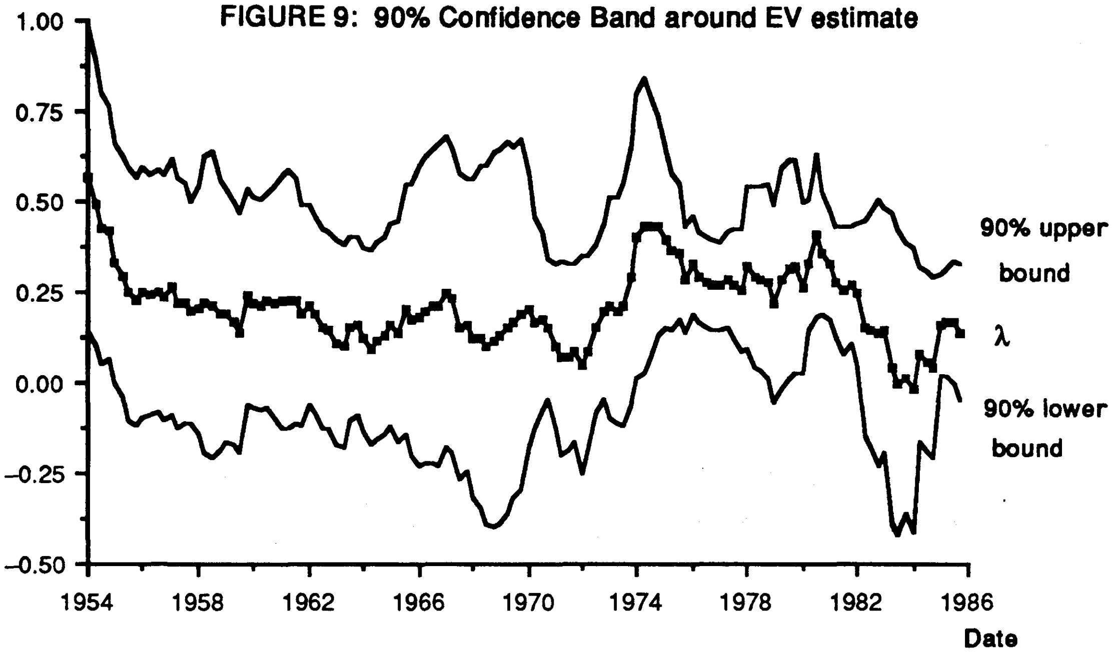
Using equation (11), the LRT statistic is 86.8, which is well above the critical value of 14.86. Thus, the null hypothesis, that the CP system performs as well as the EV in explaining consumption behavior is soundly rejected statistically. With the EV system statistically supported, the fitted values of λ_t are derived using equation (15) over the sample period of 1954:1 – 1985:4. Figure 8 plots the values of λ_t (the EV system) and λ_0 (the CP system).

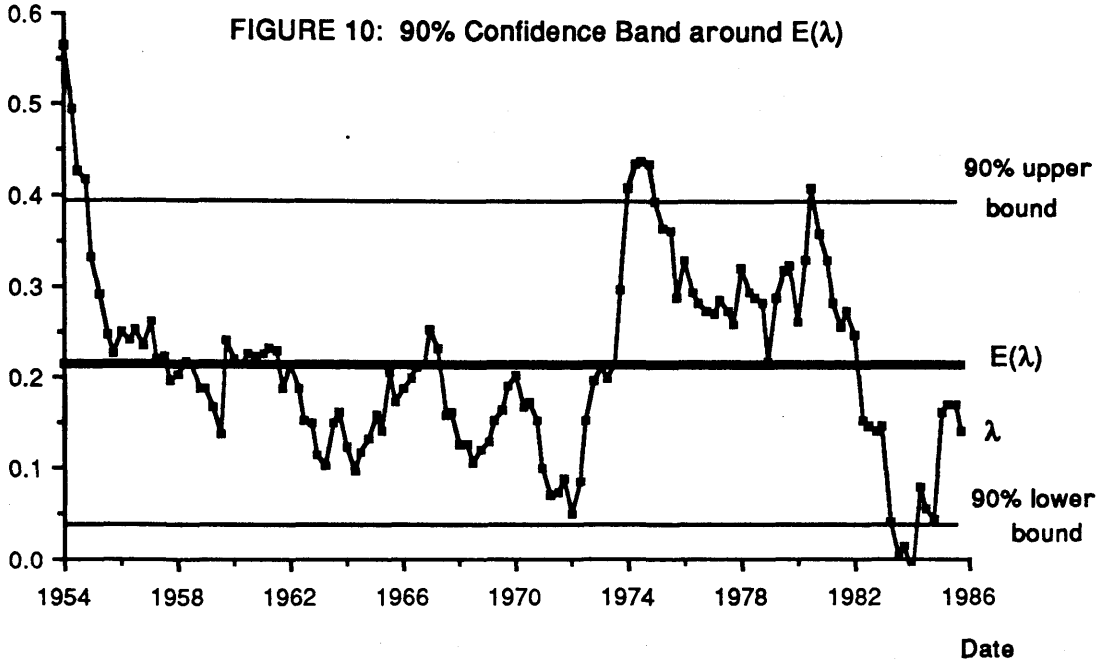


The λ_t estimates vary a great deal over the sample period. The Likelihood Ratio Test is sufficient to show that the movement in λ_t is statistically significant at the 99% confidence level. They reach a maximum value of 0.56 in 1954:1 (the first period of the sample) and approach zero in the 1983:3 – 1984:1 period. The range of these estimates is consistent with those values of λ_0 derived in previous studies. Figure 9 plots the 90% confidence band around the λ_t estimates. This indicates the degree of precision of the EV system estimates.[8] In addition, a 90% confidence band around the λ_t estimates' "in–sample" mean ($E(\lambda_t) = 0.21$) is shown in Figure 10, with a standard deviation of 0.109. In sum, it is clear that the endogenous specification of λ significantly improves the fit of the consumption equation and that its volatility is significant as well.

[8] Since λ_t is theoretically bounded between zero and one by definition, the λ_t estimates below zero but within the 90% confidence band are not economically meaningful.

FIGURE 9: 90% Confidence Band around EV estimate





In broad terms, the fraction of income which is controlled by liquidity constrained consumers is determined by both the supply of and demand for credit in financial markets. The former is primarily conditioned by the monetary and regulatory actions of the Federal Reserve as well as by commercial bank behavior, while the latter is due to the saving and consumption decisions of individuals in the economy. The explanatory variables of λ_t (for the EV system) link the policy activity of the financial market regulators and the general level of economic activity to the severity of liquidity constraints in the economy. The expansion and the coincident deregulation of credit markets can explain the general upward drift in the ratio of consumer installment loans to personal income (ic) over the sample period (see Figure 7). This is particularly evident in the 1950's, the early 1960's, and the mid–1980's. Beyond this longer term trend, "ic" is sensitive to the particular monetry – credit market regimes. For example, during the period of monetary – credit restraint in the mid–1970's and the capital controls of the first two quarters of 1980, "ic" decreased noticeably. The delinquency rate on consumer installment loans (dr) reflects, in part, the level of economic activity by gauging consumers' ability to repay debt, as well as reflecting the behavior of financial institutions regarding the riskiness of their loan activity. Tighter credit markets and a greater vigilance by financial institutions in the screening of loans tend to lower "dr", while a lower level of economic activity will diminish consumers' ability to service their debt, and so, tend to increase "dr".

For the most part, the movement in λ_t over the sample period is consistent with the developments in the credit markets and the macroeconomy. λ_t decreased from 56.4% to 22.6% over the 1954:1 to 1955:4 period. The rise in consumer expenditures relative to disposable income over this period was facilitated by a record expansion of credit, where this credit liberalization ended during 1955 as the credit markets tightened and commercial banks screened loans more carefully. The λ_t value stabilized between between 25% and 20% over the following three years, including the 1957 – early 1958 recessionary period. The economic upturn in late 1958 and 1959 saw a record rate of bank lending, and so, a decrease in λ_t of 8%. A 10% increase in λ_t occurred during 1959:4 and leveled off over the next two years in the face of an increased saving rate by consumers, characterized by a larger accumulation of liquid assets and a more restricted use of credit. A period of economic expansion typified the 1962 – 1965 period with a slight downward movement in λ_t. Over this period, monetary policy supported an expansionary fiscal policy by maintaining a ready availability of credit, thus accomodating an expansion of demand at relatively stable interest rates. During this period, there were significant changes in the character of financial instruments and in the behavior of financial institutions. In particular, new money market instruments such as the negotiable certificates of deposit (CD's) and the upward revisions in the maximum interest rates allowed to be paid on time and savings deposits enabled commercial banks to attract large inflows of such deposits. These changes allowed for a rapid increase in bank credit. Individuals were able to simultaneously increase both their borrowing and their holdings of liquid financial assets. The value of λ_t ranged from a 9.7% low in 1964:2 to a 20% high in 1965:3. A credit squeeze occurred in 1966 as the Federal Reserve sought to curb the growth of credit in the face of the extraordinary rise in credit demands in late 1965. This is reflected by an upward movement in λ_t of 6.7% from 1966:1 to 1967:1. Expansionary monetary policy (i.e., credit easing) in 1967 replaced the more restrictive policy of the previous year. λ_t reaches a trough in 1968:3 of 10.6%. This was followed by a 9.7% increase in λ_t from 1968:3 to 1970:1 during a period of monetary restraint and unusually high credit demand (caused by a high level of economic activity and high inflation expectations). λ_t reached a trough in 1972:1 of 5% (the lowest λ_t value for the 1970's) as monetary authorities followed a program of monetary easing. Credit markets tightened over the 1972 – 1975 period as higher inflation and a recessionary downturn were reflected in credit markets. A liquidity squeeze occurred during this period as banks were confronted with an inablitity to expand deposits and absorb drains on their reserves. The value of λ_t peaked in 1974:3 at 43.7%. The general decline in λ_t over 1975 – 1978 period coincided with a general decline in economic activity, presumably due to a decrease in credit demand. Monetary restraint was instituted in 1979 to check inflation by limiting money and credit growth. Moreover, credit controls were put in place during the first two quarters of 1980. Hence, there was a corresponding increase in λ_t from 21.9% in 1979:1 to a peak of 41.0% in 1980:3. Following the lifting of credit controls in 1980:3 and a postwar record rate of monetary growth in the second half of 1980, λ_t began a continuous downward path, reaching a trough below 5% in 1984. Monetary restraint returned in 1981 in an effort to curb inflation, during which the U.S. experienced a recession which lasted to November 1982. Economic expansion occurred subsequently, lasting for the remainder of the sample period. The 1980's have witnessed many changes in the institutional structure of credit markets which have enhanced the ability of financial institutions to attract funds while maintaining the stability of financial markets. Two important examples are the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Garn–St. Germain Depository Institutions Act of 1982. Among other provisions, the former mandated the phaseout of interest rate ceilings on interest–bearing accounts, while the latter authorized the money market deposit account.[9]

[9] See the *Economic Report of the President* for a review of the credit market and macroeconomic developments in the United States in each of the years represented in the sample.

5. CONCLUSION

Tests of the "Life Cycle – Permanent Income Hypothesis" have appeared in the economic literature with increasing frequency since Hall's work in 1978. In general, these studies have concluded that a significant proportion of U.S. consumption behavior does not follow that which is implied by the LC–PIH. This non–optimizing consumption behavior may be due to liquidity constraints imposed by imperfect loan markets. The magnitude of such non–optimizing behavior by consumers is still a matter of debate. Estimates of the fraction of total income held by liquidity constrained consumers (λ) vary widely, but all of the previous studies share a common thread in that they treat the value of λ as a constant over the sample period. This study has argued against the parametric specification of λ, rather asserting that λ must be specified as endogenous, determined by other economic, demographic and financial variables in the economy.

This study has estimated the volatility of the endogenous specification of λ over the 1954:1 – 1985:4 sample period, and found that the movement in λ_t is statistically significant. Moreover, as measured by the Likelihood Ratio Test, the endogenous specification of λ significantly improves the fit of the estimates the EV system is statistically supported over the CP system. This has important implications for economic policymakers, especially with respect to tax policy changes and other income redistribution programs. The results presented in this study should serve as a guide for future work on liquidity constrained consumption behavior.

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DATA APPENDIX

Population Distribution: "Population Estimates" (male and female), U.S. Dept. of Commerce, Bureau of the Census, 1948 – 1985 (annual).

Pan: all ages
Pan5: 20 –24 years
Pan6: 25–29 years
Pan7: 30–34 years
Pan9: 40–44 years
Pan10: 45–49 years
Pan11: 50–54 years

An exponential trend was used in order to impute quarterly population estimates between the annual observations. Once the quarterly population data is formed from 1946:1 thru 1985:4, the fraction of the total population in each age category is calculated:

$$f_i = \frac{pan_i}{pan}, \quad i = 5, 6, 7, 9, 10, 11.$$

where pan_i is defined above. The age distribution variable used in this study is defined as:

$$age = \log [(f_5 + f_6 + f_7) / (f_9 + f_{10} + f_{11})].$$

Personal Disposable Income: Personal disposable income is obtained from the National Income Product Accounts (N.I.P.A.) published by the U.S. government. The data was available quarterly from 1947:1 through 1987:1. It was seasonally adjusted and expressed in 1982 constant dollars.

Personal Consumption of Nondurables and Services: Nondurable and service consumption is obtained from the N.I.P.A. and was available from

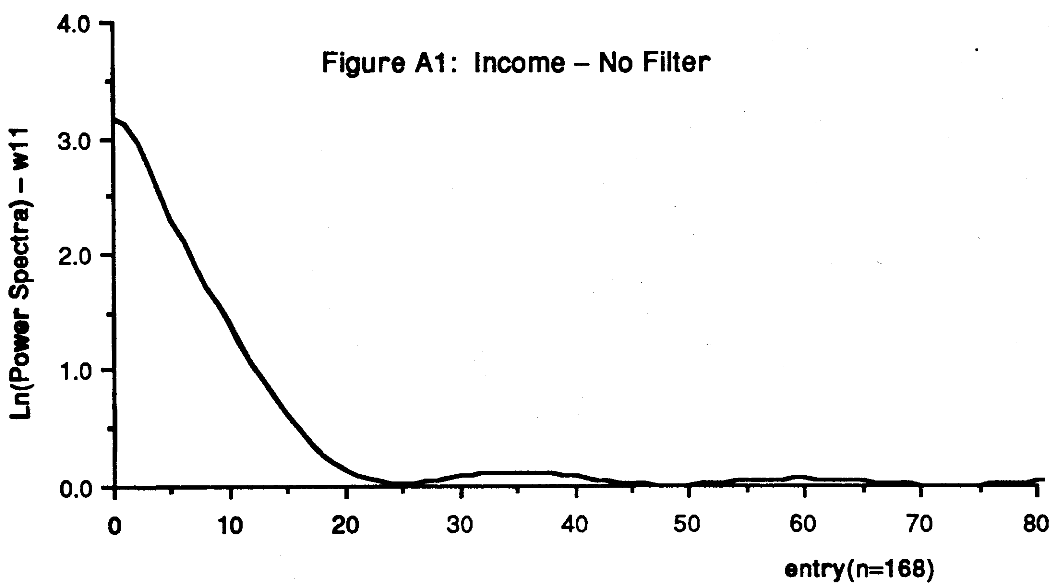
1947:1 through 1987:1. It was also seasonally adjusted and expressed in 1982 constant dollars.

Delinquency Rate on Installment Loans, 30 days and over: This data was obtained from the American Bankers Association. It is monthly data available starting in 1948:1 and is seasonally adjusted. Quarterly observations were derived from this monthly data by averaging over the three months of each quarter.

Ratio of Consumer Installment Loans to Personal Income: This is monthly data starting in 1947:1 and is seasonally adjusted. Quarterly observations were derived from this monthly data by averaging over the three month of each quarter.

APPENDIX A

The power spectra for the filters of the income series reported in Table 1 that are not contained in the main body of the text (Figures 1 – 4) are presented here. Only the income series is given here, since the consumption series follows the same pattern.



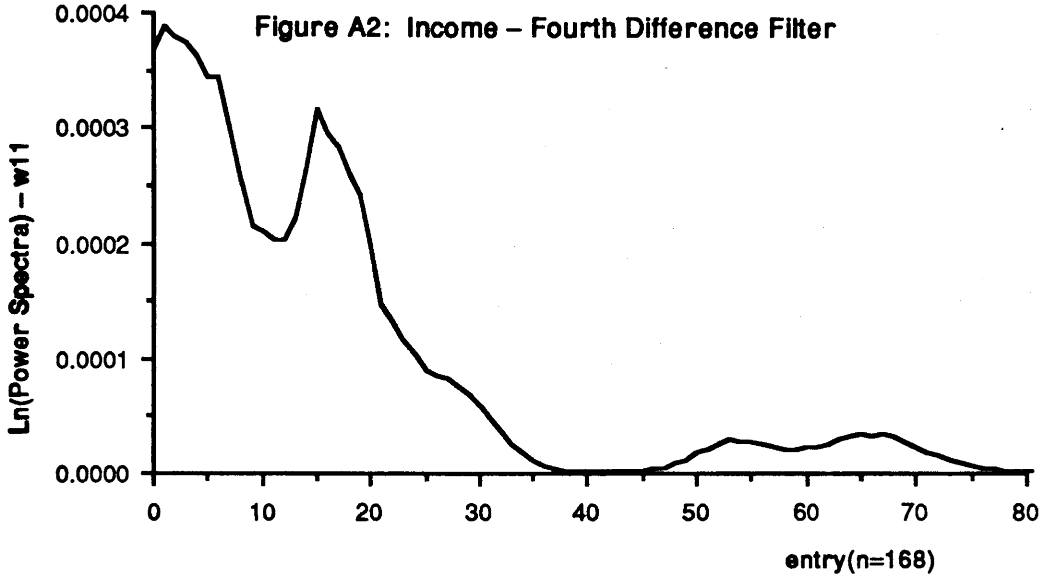


Figure A2: Income – Fourth Difference Filter
Ln(Power Spectra) – w11
0.0004
0.0003
0.0002
0.0001
0.0000
0
10
20
30
40
50
60
70
80
entry(n=168)

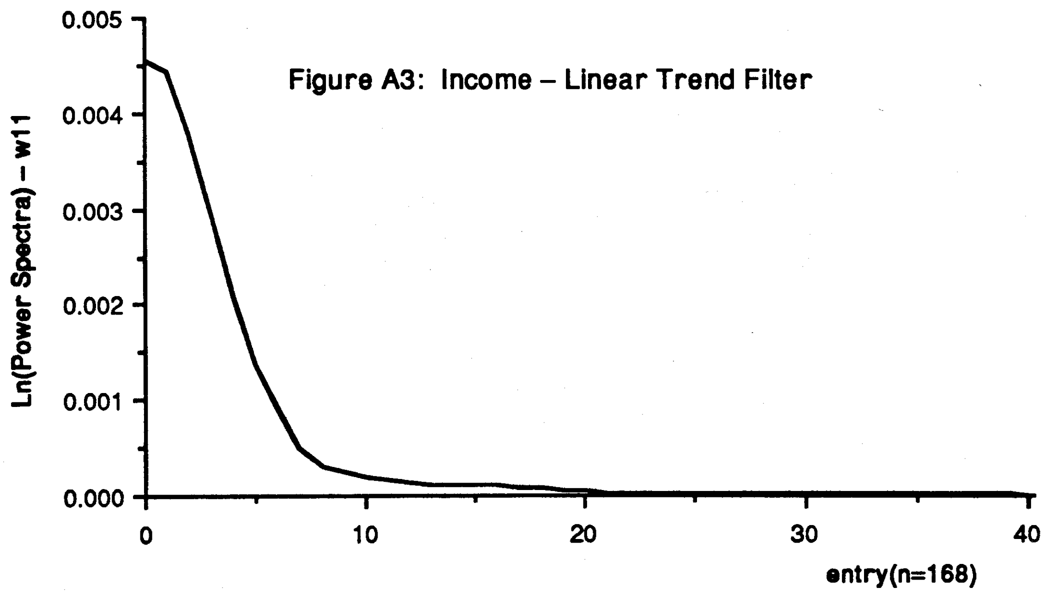


Figure A3: Income – Linear Trend Filter
Ln(Power Spectra) – w11
0.005
0.004
0.003
0.002
0.001
0.000
0
10
20
30
40
entry(n=168)

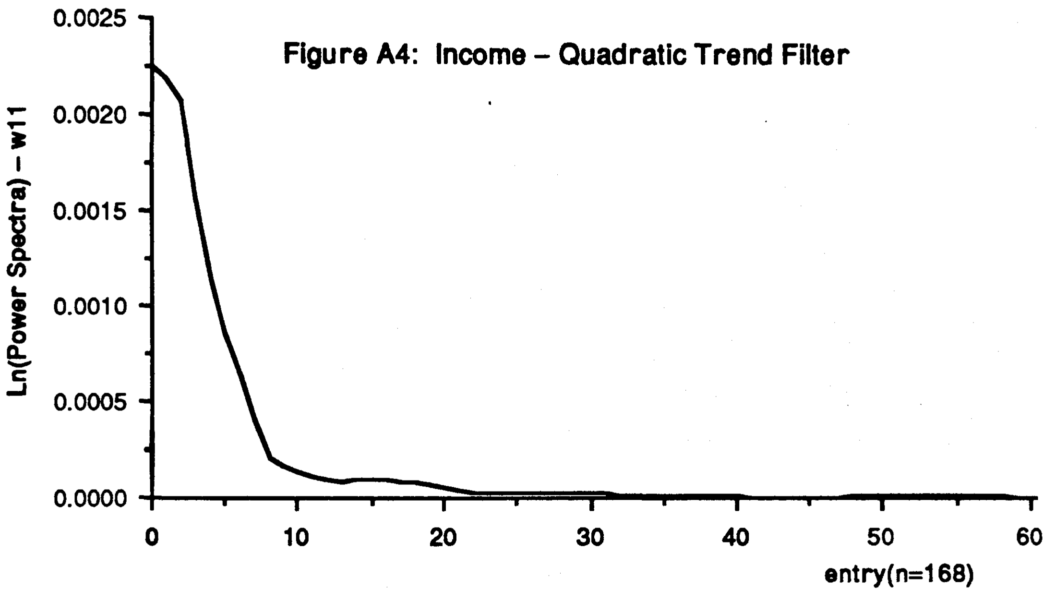


Figure A4: Income - Quadratic Trend Filter

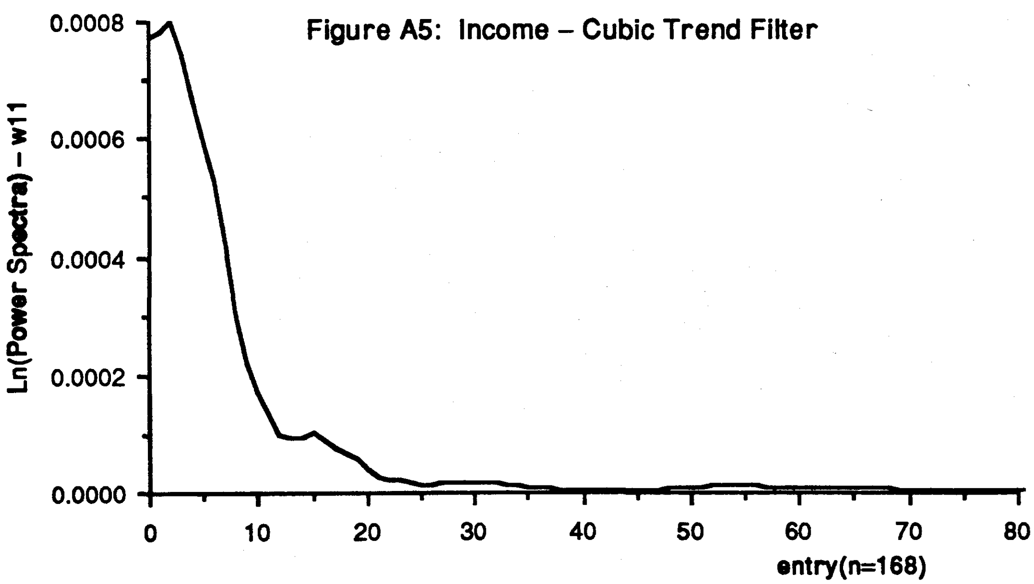


Figure A5: Income - Cubic Trend Filter